Routing of Heterogeneous Sediments over Movable Bed: Model Development

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Abstract: A one-dimensional numerical model of sediment routing is derived to simulate erosion, transport, and deposition of individual size-density fractions in the bed material within a relatively straight, nonbifurcating alluvial channel. The reach of interest is subdivided into a number of longitudinal elements of varying width-averaged properties. During each time step, flow depths and velocities in each element are determined from the gradually varied flow equation using the standard step method for backwater calculations. The bedload transport rate of each size fraction is calculated from a modified Bagnold equation implementing a novel approach that takes into consideration the effects of turbulent fluctuations in the bed shear stress. Critical shear stresses for entrainment of particles from the mixed bed are determined using relationships that treat grain protrusion and hiding. The transport of particles in suspension is modeled using a convection-diffusion sediment continuity equation, either explicitly solved by the Rouse equation or implicitly solved using a finite difference scheme. Changing vertical geometry is handled using coördinate stretching. The velocity profile in the vertical is calculated at each point using the von Karman-Prandtl logarithmic velocity distribution, and vertical sediment diffusivities for the suspended sediment are computed assuming a parabolic distribution for diffusion of fluid momentum. Interaction of the transported load and the bed is calculated by a bed-continuity equation solved for each size-density fraction in an active layer. Bed composition and elevation are monitored through time, so that simulations produce complete stratigraphic sequences of sediment. The advantages of this model over previous models are a treatment of turbulent fluctuations of bed shear stress, minimization of calibration factors, and explicit consideration of multiple grain densities.

Introduction

Problems such as bed degradation and armoring downstream from dams, ...voir sedimentation, and sorting of dense minerals in placers require a ...atment of quasi-unsteady, nonuniform open channel flow over an erod-..., heterogeneous bed. The transport rates of each size fraction of every ...al density in transport must be known as a function of longitudinal ...sition and time, which in turn requires knowledge of the flow hydraulics ... size-density composition of the bed. As particles are eroded from or ...posited onto the bed, the bed's size-density composition may change, ...reby altering both the flow hydraulics and fractional transport rates. As ...inted out by Rahuel et al. (1989), previous attempts to model these ...ocesses [e.g., Alonso et al. (1981), Karim et al. (1987), Lee and Odgaard ...986), Lu and Shen (1986), and Park and Jain (1987)] understandably have ...ered from simplified treatments of sediment transport and hydraulic ...ting of sediment mixtures.

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...ote. Discussion open until July 1, 1992. Separate discussions should be submitted ...he individual papers in this symposium. To extend the closing date one month, ...itten request must be filed with the ASCE Manager of Journals. The manuscript ...his paper was submitted for review and possible publication on March 15, 1991. ... paper is part of the *Journal of Hydraulic Engineering*, Vol. 118, No. 2, February, ... ©ASCE, ISSN 0733-9429/92/0002-0246/$1.00 + $.15 per page. Paper No. 1551.

A one-dimensional numerical model of sediment routing is presented here to address some of these difficulties. This model investigating density and size sorting (MIDAS) simulates erosion, transport, and deposition of individual size-density fractions in the bed material available within a relatively straight, nonbifurcating alluvial channel. To keep the computations simple, an uncoupled steady solution of fluid and sediment transport equations is used during each time step. The reach of interest is subdivided into a number of longitudinal elements (nodes) of varying width-averaged properties. During each time step, flow depths and velocities in each node are determined from the gradually varied flow equation using the standard step method for backwater calculations. The bedload transport rate of each size-density fraction is calculated from a modified Bagnold equation (Bagnold 1973; Bridge and Dominic 1984; Engelund and Fredsoe 1976) implemented using a novel approach that takes into consideration the effects of temporally short-scale turbulent fluctuations in the bed shear stress. Critical shear stresses for entrainment of particles in the mixed bed are determined using relationships of Komar (1987a, 1987b, 1989), Egiazaroff (1967), or James (1990). The transport of particles in suspension is modeled using a convection-diffusion sediment continuity equation, either explicitly solved by the Rouse equation or implicitly solved using a finite difference scheme. Changing vertical geometry is handled using coordinate stretching. The velocity profile in the vertical is calculated at each node using the von Karman-Prandtl logarithmic velocity distribution. Vertical sediment diffusivities for the suspended load are computed assuming a parabolic distribution for diffusion of fluid momentum. Interaction of the transported load and the bed is calculated by a bed-continuity equation solved for each size-density fraction in an active layer. The advantages of this approach over previous models are a treatment of turbulent fluctuations of bed shear stress, minimization of calibration factors, and explicit consideration of multiple grain densities.

Flow Model

Mean flow velocity and depth at a node, nd, are obtained from the gradually varied flow and conservation of mass equations written in the coordinate system of Fig. 1

$$\frac{d}{dx}(Q\bar{V}_x) + gA\frac{d}{dx}(d) = gA(S_0 - S_f) \quad \text{(1)}$$

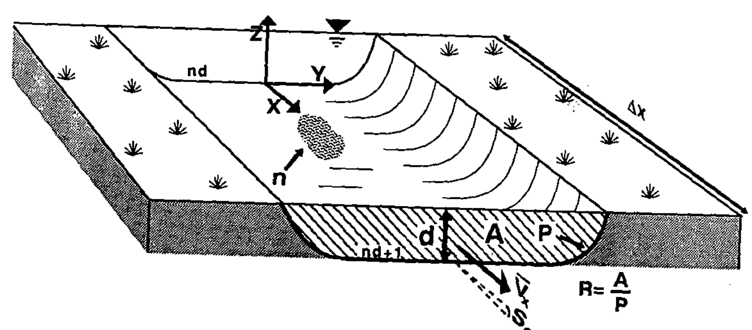

FIG. 1. Schematic Representation of Gradually Varied Flow Model

$$Q = \overline{V}_x A \quad (2)$$

in which Q = water discharge; x = the downstream distance along the channel; $\overline{V}_x$ = the cross-sectionally averaged longitudinal flow velocity; g = the gravitational acceleration; A = the cross-sectional area; d = the flow depth; S_0 = the bed slope; and S_f = the friction slope. Friction slope is obtained from Manning's equation

$$S_f = \frac{n^2 \overline{V}_x^2}{k_M^2 R^{4/3}} \quad (3)$$

where n = the Manning roughness coefficient (expressed in $m^{1/6}$), constant throughout a run; k_M = 1 $m^{1/2}/s$; and R = the hydraulic radius (m).

Water discharge, bed elevations, and cross-sectional areas at each node are specified as input at the start of a simulation, after which they evolve according to the equations. Flow equations (1) and (2) are completed by specifying one of two boundary conditions: a fixed flow depth at any node in the channel or a constant water surface elevation at the downstream end of the reach. It is assumed that the flow will always remain subcritical, as is the case in most naturally occurring river systems. When flow becomes supercritical, the flow conditions are approximated by the critical conditions.

It may be argued that this simple treatment of the flow is inadequate. For example, more advanced approaches for calculating S_f in (3) (Brownlie 1983; White et al. 1987) are available. This first version of MIDAS was intentionally kept simple, so that the contribution of each component could be more fully understood. The second part of this article (Vogel et al. 1992) shows that reasonable results can be obtained with this simplified approach.

Determination of Bed Shear Stresses and Shear Velocities

The skin friction component of the temporal mean shear velocity u'_* is used as a measure of the force available to transport sediments. Einstein (1950) and Einstein and Barbarossa (1952) proposed that the depth-averaged velocity at a cross section for a rough boundary is given by

$$\frac{\overline{V}_x}{u'_*} = 5.75 \log_{10}\left(\frac{12.27}{\frac{k_s}{R'}}\right) \quad (4)$$

where k_s = the grain roughness, assumed to be 2.5 times the median size of bed particles; and R' = that portion of the hydraulic radius attributable to skin friction. Shear velocity is calculated from the DuBoys (1879) equation

$$u'_* = \sqrt{\frac{\overline{\tau}'}{\rho}} = \sqrt{gR'S_f} \quad (5)$$

in which $\overline{\tau}'$ = the skin friction component of the temporal mean bed shear stress hereafter called the *effective bed shear stress*; and ρ = the fluid density. The average velocity and friction slope are obtained from (1)–(3), thereby allowing (4) and (5) to be solved iteratively for R' and thus u'_* and $\overline{\tau}'$.

Shear Stress Distribution

It has long been established that flow turbulence results in a fluctuation of local instantaneous bed shear stresses about a mean value (Blinco and Simons 1974; Cheng and Clyde 1972; Christensen 1972; Grass 1970). Because the modes and rates of sediment transport vary as a function of turbulence (Grass 1982; Thorne et al. 1989), it is desirable to incorporate a distribution of instantaneous bed shear stresses into a bed-load transport model. Observations suggest that for fully turbulent flow, the instantaneous bed shear stresses measured in time at a location (Fig. 2) approximately follow a Gaussian distribution $f(\tau')$, with a coefficient of variation equal to 0.4 (Bridge 1981)

$$f(\tau') = \frac{1}{\sigma_\tau \sqrt{2\pi}} e^{-1/2[(\tau' - \overline{\tau}')/\sigma_\tau]^2} \quad (6)$$

where τ' = the instantaneous effective bed shear stress; and σ_τ = the standard deviation of the instantaneous bed shear stress distribution. For a hydraulically smooth flow, the distribution becomes more positively skewed as a function of decreasing boundary Reynolds number (Blinco and Simons 1974; Grass 1970) and is better defined by the logarithm of (6) or by a gamma function.

In the present model, the bedload transport rate of each size fraction is calculated for each of N bed shear stress ranges using either (6) or its logarithmic equivalent. Each range has a mean value of τ'_k, and a width $\Delta\tau'$

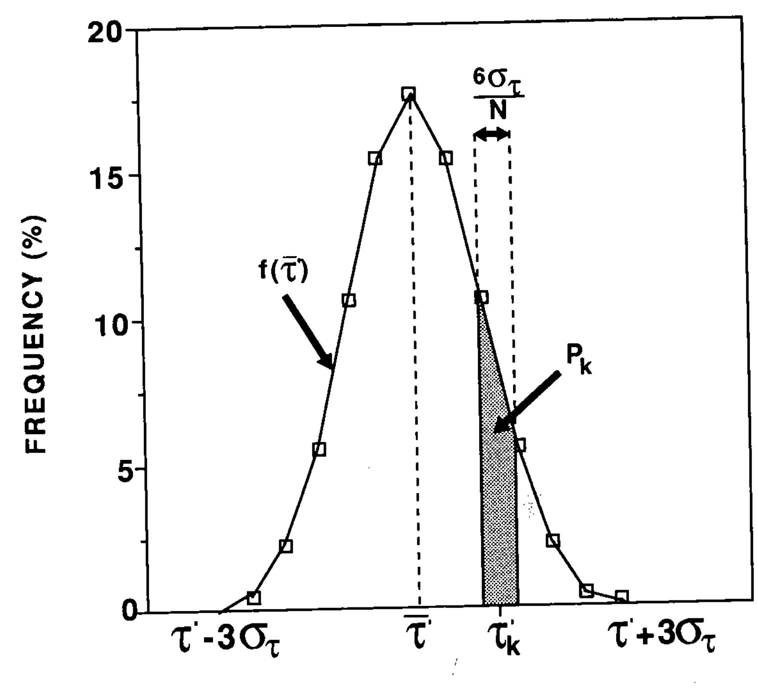

FIG. 2. Example of Discretized, Instantaneous Bed Shear Stress Distribution, with Mean Value of $\overline{\tau}'$ and Standard Deviation of σ_τ

of $6\sigma_\tau/N$. The proportion of time allotted to each shear stress range P_k, is obtained by integrating the distribution function over the interval $\Delta\tau'$.

Bedload Transport

By bedload we mean here all those sliding, rolling, and saltating grains supported at least in part by collisions with other grains or contact with the bed. In water, the grains travel within a few grain diameters of the bed as a low-concentration, dispersed, grain flow. Grains are considered to be in motion when

$$\Theta_{0k} > \Theta_{cij} \quad (7)$$

in which Θ_{0k} = the dimensionless bed shear stress of the kth shear stress interval; and Θ_{cij} = the critical Shields parameter for the ith size and jth density grain fraction.

Operationally, grains of fall velocity w_{ij} are considered to be in bedload transport under bed shear stress interval k when

$$w_{ij} \geq Bu'_{*k} \quad (8)$$

where u'_{*k} = the effective shear velocity of the kth shear stress interval; and B ranges from 1.25 (Bagnold 1973) to 0.8 (Engelund and Fredsoe 1976) and here is given the value 0.8 (Bridge 1981).

Fall velocities for nonspherical particles w_{ij} are obtained from the empirical equation of Dietrich (1982)

$$\log W_* = -3.76715 + 1.92944(\log D_*) - 0.09815(\log D_*)^2$$

$$- 0.00575(\log D_*)^3 + 0.00056(\log D_*)^4 \quad (9)$$

$$W_* = \frac{\rho w_{ij}^3}{(\sigma_j - \rho)g\nu} \quad (10)$$

$$D_* = \frac{(\sigma_j - \rho)gD_{nij}^3}{\rho\nu^2} \quad (11)$$

where W_* = the dimensionless settling velocity; D_* = the dimensionless grain diameter; σ_j = the mineral density of the jth fraction; ν = the kinematic viscosity; and D_{nij} = the nominal diameter of the ith grain size fraction and the jth mineral density fraction. In applying the Dietrich equation in the model, it is assumed that the nominal diameter for a particle can be represented by the grain size derived from sieve measurements and that the fall velocity of a particle in a turbulent flow is the same as its fall velocity in a quiescent fluid.

Bedload Formula

Of the many formulas predicting the bedload transport rate [Ackers and White (1973), Bridge and Dominic (1984), Einstein (1950), Meyer-Peter and Muller (1948), van Rijn (1984a); and Yang (1973), to name but a few], a modified Bagnold equation (Bridge and Dominic 1984; Engelund and Fredsoe 1976) was selected because: (1) It is a physically based equation with experimentally determined constants; (2) the transport rate of each grain size can be determined separately; (3) its parameters are easily calculated and well suited to numerical modeling using shear stress intervals; and (4) it has been shown to fit natural data very well (Bridge and Dominic 1984).

The bedload transport equation is

$$i_{b_{ijk}} = F_{ij}P_k \frac{h}{\tan\alpha}(u'_{*k} - u_{*cij})(\tau'_k - \tau_{cij}) \quad (12)$$

in which $i_{b_{ijk}}$ = the bedload transport rate (immersed weight transported per unit width per unit time) of the ith size interval and jth density grains due to the kth instantaneous shear stress interval; F_{ij} = the volumetric proportion of the ith-jth fraction in the active layer; P_k = the proportion of time the kth shear stress is active; h = a parameter equal to $1/\kappa \ln(z/k)$, where κ = von Karman's constant, z = the distance from the bed to the center of fluid thrust on bedload grains, and k = the bed roughness; $\tan\alpha$ = a dynamic friction coefficient; and u_*c_{ij} and τ_{cij} = the critical shear velocity shear stress necessary to entrain the ij size-density combination. Originally, Bridge and Dominic claimed $h/\tan\alpha$ was weakly a function of excess shear velocity and grain fall velocity. Recently however, Bridge (personal communication, 1988) has suggested that a constant value of 10 may be more appropriate for lower stage plane beds and dune backs, while a value of 17 is a reasonable average for upper stage plane beds. This is used in the present formulation.

Reasonable values of h range from 6.8 to 8.5 for hydraulically rough flows (Bridge and Bennett 1991). Unlike some previous formulations, volumetric proportion is used in (12), because the probability of a grain fraction being available for transport is proportional to its volume in the active layer. To obtain the total transport rate of the ith-jth fraction, (12) must be summed over all instantaneous bed shear stresses for which the fraction is in motion as bedload

$$i_{b_{ij}} = \sum_{k=e}^{f} i_{b_{ijk}} \quad (13)$$

where e = the interval of smallest instantaneous shear stress greater than τ_{cij}; and f = the interval of largest instantaneous shear stress less than or equal to the shear stress necessary to suspend the grain.

Critical Shear Stress

Accurate prediction of bedload transport depends strongly upon the critical shear stress necessary to initiate motion of a specific size-density particle on a heterogeneous bed. The Shields relationship (1936) is clearly inappropriate for grains much larger or smaller than the medium size, because the relationship does not account for relative protrusion and grain hiding effects (Fenton and Abbott 1977; Komar 1987a, 1987b, 1989). Recent entrainment work (Andrews 1983; Egiazaroff 1967; James 1990; Komar 1987a, 1987b, 1989; Parker et al. 1982; Slingerland 1977) has focused on determining the relationship between a grain's critical Shields parameter Θ_{cij} and the ratio of its grain diameter D_{ij} to the medium grain diameter D_{50}. MIDAS allows selection of one of three different formulations (Egiazaroff 1967; Komar, 1989; James 1990), which were chosen for incorporation into the model based on their physical derivation and their experimental verification against large data sets.

The first, the modified Egiazaroff entrainment equation for mixed sizes ($0.3 < D_{ij}/D_{50} < 10$) under hydraulically rough flows, is

$$\Theta_{c_{ij}} = \Theta_{c50} \frac{\log^2(19)}{\log^2\left(19 \dfrac{D_{ij}}{D_{50}}\right)} \quad \text{(14)}$$

in which Θ_{c50} = the critical Shields parameter for the median size; D_{ij} = the grain size of the ith size fraction of the jth density species; D_{50} = the median bed grain size; and

$$\Theta_{c_{ij}} = \frac{\tau_{c_{ij}}}{(\sigma_j - \rho)gD_{ij}} \quad \text{(15)}$$

where $\tau_{c_{ij}}$ = the critical shear stress necessary to entrain grains of the ith size and jth density interval.

The second, the James entrainment function, is

$$\Theta_{c_{ij}} = \frac{k_1k_2\sin(\phi - \beta)}{16.5311\left[\log^2\left(30.2 \cdot k_9 \dfrac{x'd}{k_{10}} k_s\right)\right][C_Dk_5k_6(k_2\cos\phi + k_3) + C_Lk_7k_8(k_2\sin\phi + k_4)]} \quad \text{(16)}$$

where k_1–k_{10} = parameters defined by James (1990); C_D and C_L = the coefficients of drag and lift, respectively; ϕ = the grain pivot angle; β = the bed slope angle; and x' = the velocity profile correction factor, which has a value of 1.0 for hydraulically rough flow (James 1990).

The third entrainment function, Komar's entrainment function, is valid over the range $0.3 < D_{ij}/D_{50} < 22$, and can be formulated as (Komar 1989)

$$\Theta_{c_{ij}} = \Theta_{c50}\left(\frac{D_{ij}}{D_{50}}\right)^{-m} \quad \text{(17)}$$

A matter of recent dispute has been the appropriate value of m in (17) (Andrews 1983; Komar 1987b; Parker et al. 1982). Recently, Komar (1989) has defended his choice of m = 0.65, indicating that Andrews' sediment collection method may not have captured the largest entrained clasts, while Parker et al. were examining relative transport rates using (17) as a normalization equation for their transport relationships.

Eqs. (14)–(17) were originally developed for single density sediments. It is assumed that they can be applied to sediments with multiple density fractions. Note that for very small D_{ij}/D_{50}, the critical shear stress of grain D_{ij}, as determined by (14) or (16), becomes infinite.

The critical Shields parameter of the median size Θ_{c50} is assumed to be accurately predicted by the Shields curve under the assumption that a grain of the medium size has the greatest probability to rest on grains of nearly equal size. The Shields curve is approximated by three equations in the manner of Bridge (1981)

$$\Theta_{c50} = 0.1(\mathrm{Re}_D)^{-0.3}, \quad \text{for } \mathrm{Re}_D < 1 \quad \text{(18)}$$

$$\ln(\Theta_{c50}) = -2.26 - 0.905\ln(\mathrm{Re}_D) + 0.168\ln^2(\mathrm{Re}_D),$$

$$\text{for } 1 < \mathrm{Re}_D < 60 \quad \text{(19)}$$

$$\Theta_{c50} = 0.045, \quad \mathrm{Re}_D > 60 \quad \text{(20)}$$

$$\mathrm{Re}_D = \frac{u_* D_{50}}{\nu} \quad \text{(21)}$$

in which Re_D = the grain Reynolds number.

Suspended Load Transport

The suspended load is the width- and depth-integrated distribution of suspended solids carried forward by a two-dimensional turbulent flow. As in other formulations, it is assumed that the boundary between the bed-load and suspended load occurs at the top of the moving bed layer, a distance $z = a$, from the reference plane (Fig. 3).

The moving bed layer thickness a can be calculated from any number of formulations and in the model is computed using either the relationship of Einstein (1950)

$$a = 2D_{50} \quad \text{(22)}$$

or the relationship of Bridge and Dominic (1984)

$$\frac{a}{D_{50}} = 2.53(\Theta_{0k} - \Theta_{c_{ij}}) + 0.5 \quad \text{(23)}$$

where Θ_{0k} = the dimensionless bed shear stress using τ'_k and D_{50}. These equations for moving bed layer thickness were derived for beds with uniform sediment sizes, and their applicability to the case of individual size density fraction needs further investigation.

Operationally, grains are considered to be in suspension when (7) is true, but (8) is false. The suspended sediment concentrations for the individual size-density fractions are calculated from the convection-diffusion equation

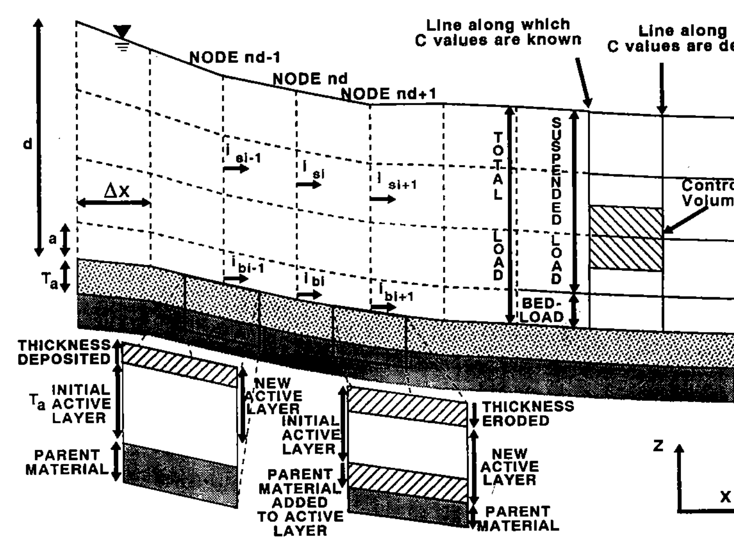

FIG. 3. Schematic Representation of Grid Used to Calculate Suspended Load and of Treatment of Bed after Erosion and Deposition

for the vertical distribution of suspended sediment. This is derived from a mass balance for suspended sediment by assuming that steady state conditions apply over a time step, that the dependent variables are constant in a cross-stream (y-) direction, and that the x-directed diffusivity is negligible. The equation is

$$\frac{\partial}{\partial x}[V_x(z)C(z)_{ij}] + \frac{\partial}{\partial z}[(V_z - w_{ij})C(z)_{ij}] = \frac{\partial}{\partial z}\left[\varepsilon_s \frac{\partial}{\partial z}(C(z)_{ij})\right] \quad \ldots\ldots (24)$$

in which $C(z)_{ij}$ = the local suspended sediment concentration (total immersed mass of solids of the ith-jth fraction per unit volume of fluid); V_z = the local, temporally averaged vertical fluid velocity; and ε_s = the sediment eddy diffusion coefficient in the vertical. The local vertical velocity V_z is considered to be insignificant.

Fluid and Sediment Momentum Transfer Components

Solution of the equations describing the vertical distribution of suspended sediment requires an accurate determination of flow velocity and turbulent diffusivity in the vertical. These in turn depend upon such factors as uniformity and steadiness of flow, bed friction (which is dependent on the flow regime), and the presence of suspended sediment. Here the approach of Einstein and Barborossa (1952) is adopted, although the accuracy of this approach under nonuniform flow conditions needs to be verified. Under uniform flow conditions, the von Karman-Prandtl mixing length model of turbulence describes the vertical distribution of velocity and turbulent diffusivity quite well (Einstein 1950; Einstein and Barborossa 1952). If the bed roughness is much smaller than the flow depth, the resulting logarithmic velocity distribution, written in terms of the cross-sectionally averaged flow velocity $\overline{V}_x$ is

$$V_x(z) = \frac{u_*}{\kappa}\left[\log\left(\frac{z}{d}\right) + 1\right] + \overline{V}_x \quad \ldots\ldots (25)$$

in which $V_x(z)$ = the longitudinal velocity as a function of depth; and d = the total flow depth. Following Middleton and Southard (1984), the sediment eddy diffusion coefficient ε_s is assumed to equal the fluid eddy diffusion coefficient in the vertical ε_z. If a linear distribution of shear stress is assumed in the fluid, then the vertical distribution of the eddy diffusion coefficient can be written as

$$\varepsilon_z = \kappa u_* \frac{z}{d}(d - z) \quad \ldots\ldots (26)$$

Although this is the approach used, it should be noted that van Rijn (1984b), using the results of Coleman (1970), argues for a constant turbulent diffusivity in the upper half of the channel depth

$$\varepsilon_z = \varepsilon_{max} = \frac{1}{4}(\kappa u_* D) \quad \ldots\ldots (27)$$

This result is contradicted by the experiments of Ueda et al. (1976) and Nezu and Rodi (1986). Van Rijn (1984b) suggests further modifications to take into consideration the effects of nonuniformity in the flow. A simple first-order differential equation due to van Rijn can be used to adjust the

maximum eddy viscosity under nonuniform flow conditions, although the van Rijn (1984b) model to date is calibrated solely for flow over trenches dredged perpendicular to the flow direction.

Equilibrium Approximation

For reaches approximated by long distance steps ($d/\Delta x$ very small), the horizontal gradient term, $\partial/\partial x[V_x(z)C(z)_{ij}]$ becomes negligible, in which case integration of (24) yields the Rouse (1937) equation

$$\frac{C(z)_{ij}}{C(a)_{ij}} = \left(\frac{D - z}{z}\frac{a}{D - a}\right)^{(w_{ij}/\kappa u_*)} \quad \ldots\ldots (28)$$

in which $C(z)_{ij}$ and $C(a)_{ij}$ = the concentrations of the ith-jth fraction at flow heights z and a, respectively; and u_* = the total, time-averaged shear velocity (not just the skin-friction component). Eq. (28) results from a linear shear stress distribution and a logarithmic velocity distribution, and assumes that the concentrations are not affected by turbulent fluctuations in τ.

The reference concentration at the height of the moving bed layer $C(a)_{ij}$ is calculated by assuming that grains in suspension have a concentration in the moving bed layer predicted by the Bridge and Dominic equation (12)

$$C(a)_{ij} = \frac{i_{b_{ij}}}{U_{b_{ij}} a g} \quad \ldots\ldots (29)$$

in which $C(a)_{ij}$ = the concentration of the ith-jth size-density fraction in the moving bed layer; $U_{b_{ij}}$ = the bed-load grain velocity of the ith-jth size-density fraction; and a = the thickness of the moving bed layer.

Bridge and Dominic (1984) derive an expression for $U_{b_{ij}}$

$$U_{b_{ij}} = h(u_* - u_{*c_{ij}}) \quad \ldots\ldots (30)$$

Finally, the suspended load transport rate of a grain size-density fraction through a cross section per unit width, i_s, is the depth-integrated suspended load flux

$$i_{s_{ij}} = \int_a^d V_x(z)C(z)_{ij}dz \quad \ldots\ldots (31)$$

Nonequilibrium Approximation

For short and rapidly varying reaches, (28) is inappropriate, and (24) must be solved in full, as discussed in the following. In this case, (22), (23), (29), and (30) are used to determine the boundary values.

Treatment of Bed

In previous studies (Alonso et al. 1981; Bennett and Nordin 1977; Karim et al. 1987; Lee and Odgaard 1986; Park and Jain 1987; Rahuel et al. (1989), the bed region is divided into two or three horizons: (1) An upper zone, termed the mixing layer, representing the space occupied by dunes and ripples; (2) a top horizon of the mixing layer, termed an active layer, in which continous exchange of sediment particles between the bed and flow takes place; and (3) the subjacent static bed. As pointed out by Rahuel et al. (1989), the distinction between an active and mixing layer is somewhat artificial, depending as it does on the time scale of consideration. At time scales shorter than it takes for a bedform to traverse its own wavelength, the two are identical.

In MIDAS only an active layer is defined (Fig. 3). The active layer, T_a, is considered to consist of j well-mixed density fractions, each with its own size distribution. Each size class is represented by a median diameter D_{ij}. Particle exchange occurs between the active and moving bed layers during each time step, after which the particle size-density distribution in the active layer is updated to allow for any erosion or deposition of the different size-density fractions (Fig. 3). If net degradation occurs during the time step, the active layer is replenished from the underlying parent bed material by an amount equal to the thickness of sediment eroded (Fig. 3). If net deposition occurs, the base of the active layer moves up by an amount equal to the thickness of deposited material (Fig. 3).

Previous authors have considered the active layer thickness to be a function of dune height or, in the absence of dunes, water depth (Lee and Odgaard 1986; Rahuel et al. 1989), or of particle size (Borah et al. 1982; Park and Jain 1987). In MIDAS, it is defined from sensitivity experiments as

$$T_a = 2D_{50} \frac{\bar{\tau}'}{\tau_{c50}} \qquad (32)$$

This formulation seems reasonable because active layer thickness should increase with increasing excess shear stress.

Computation of the sediment mass exchange between the flow and the active layer, and consequently, computation of erosion and deposition at each downstream site, is accomplished using the conservation of mass equation written for each size-density fraction. It is assumed that the time step has been chosen sufficiently small such that fluid flow and sediment transport rates may be considered constant over the time step. For steady flows, the sediment continuity equation for a size-density interval expressed in terms of width-integrated sediment discharge is

$$(1 - p) \frac{\partial}{\partial t} (b z_{b_{ij}}) + \frac{1}{(\sigma_j - \rho)} \left[\frac{1}{g} \frac{\partial}{\partial x} (b i_{b_{ij}}) + \frac{\partial}{\partial x} (b i_{s_{ij}}) \right] = 0 \qquad (33)$$

in which p = bed porosity; b = the width of the active bed, assumed to be equal to the flow width; z_{bij} = bed elevation attributable to the ith-jth bed fraction; and i_{bij} and $i_{s_{ij}}$ are the bedload (kg s^{-3}) and suspended load (kg s^{-1} m^{-1}) transport rates of the ith-jth fraction as defined in (13) and (31). If the theoretical transport rate of a certain size-density fraction exceeds the amount available in the active layer, the calculated bedload and suspended load transport rates of that size-density fraction are reduced until the amount eroded just equals the amount available. If the theoretical transport rate of every size-density fraction is exceeded, the entire active layer is eroded.

Solution of Equation Set

Solution of the equation set proceeds according to the flow diagram of Fig. 4. The reach of interest is discretized into a finite number of nodes, Δx apart, at which hydraulic geometries and bed size-density distributions are known. At the start of a new time step, the gradually varied flow equations [(1)–(3)] are solved using the standard step method of Henderson

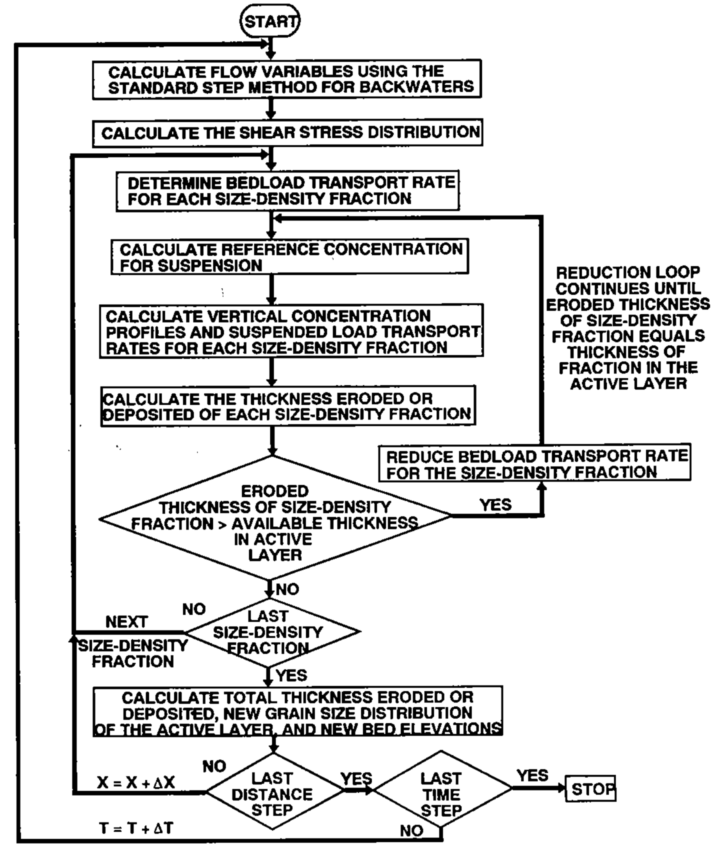

FIG. 4. Flow Chart of Model

(1966), subject to a water surface elevation at the downstream boundary node. Next, the skin friction component of bed shear stress is calculated at each node via (4) and (5). A distribution of instantaneous bed shear stresses and the momentum transfer coefficients for use in the suspended load calculations are computed from (6) and (26), respectively.

Next, the critical shear stresses for suspension of each size-density fraction in the active layer at each node are calculated from (8)–(11), and the critical shear stresses for entrainment are determined using (14)–(17). Bedload transport rates of each fraction at each node are calculated via (12) and (13). This provides the reference concentrations necessary to calculate the suspended loads from (28)–(31).

In the equilibrium case the suspension profile is calculated from (28). In the nonequilibrium case (24) is integrated over the flow width and is solved using a finite-volume formulation, whereby the suspension region is divided into a number of quadrilateral volumes as shown in Fig. 3, with equal spacing of nodes in the longitudinal direction and a constant number of nodes in the vertical. This solution procedure deals with complicated flow geometry without the necessity of writing the equation in curvilinear coordinates, while preserving the property of conservation. The suspension region is considered to extend from the top of the moving bed layer to the free water surface, the two points where boundary conditions are specified (Fig. 3). At every grid point in the vertical, the values of $C(z)_{ij}$ are calculated at the discrete z/D value for horizontal grid x-value. The new values of $C(z)_{ij}$ are then calculated for the same z/D value at the next horizontal grid point. By stepwise repetition of this basic operation, the whole field of interest is covered. The difference equations in the existing model express the mean values of each of the terms in (24) in terms of the unknown values of $C(z)_{ij}$ downstream and the known values of $C(z)_{ij}$ upstream. Linear variations of $C(z)_{ij}$ over a grid element are assumed in both the x- and z-directions, corresponding to a Crank-Nicholson procedure. The resulting equation set can be reduced to and solved as a tridiagonal matrix at each step in the marching procedure. Eq. (31) is then solved for each node using the calculated concentration values.

Next, the bed continuity equation (33) is solved for each size-density fraction at each node, nd, using a modified Preissmann scheme (Lyn and Goodwin 1987), which has been shown to be both accurate and stable for this application

$$\Delta(bz_{b_{ij}})_t = \frac{\Delta t}{\Delta x(\sigma_j - \rho)(1 - p)}\left(\left\{\Phi\left[\Delta\left(\frac{b}{g}\, i_{b_{ij}} + bi_{s_{ij}}\right)_{nd+1}\right]\right\} + \left\{(1 - \Phi)\left[\Delta\left(\frac{b}{g}\, i_{b_{ij}} + bi_{s_{ij}}\right)_{nd}\right]\right\}\right) \quad (34)$$

in which $\Delta\ (\text{variable})_{\text{subscript}} = (\text{variable})_{\text{subscript}} - (\text{variable})_{\text{subscript-1}}$; and Φ = a weighting factor between zero and one. In the model, Φ is set to 0.55, as this reasonably approximates a central difference scheme ($\Phi = 0.5$) without its inherent stability problems. Finally, at each node, new proportions of each size-density fraction of the active layer and the bed elevation are recomputed, after which the computation proceeds to the next time step. In the case of net erosion, the new active layer will be a composite of the remaining active layer material and of the underlying material (Fig. 3). In the case of net deposition, the new active layer will be a composite of the remaining active layer material and of the deposited material (Fig. 3).

Conclusions

A one-dimensional sediment routing model has been developed for simulating water and sediment transport of heterogeneous size-density material within a relatively straight, nonbifurcating alluvial channel. Its advantages over previous models are a treatment of turbulent fluctuations of bed shear stress, minimization of calibration factors, and explicit consideration of multiple grain densities.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = cross-sectional flow area (m^2);
a = height of moving bed layer (m);
B = empirical constant used in suspension inequality;
b = flow width (m);
C_D = coefficient of drag;
$C(z)_{ij}$ = local suspended sediment concentration at height z of grain size i and mineral density j (kg/m^3);
C_L = coefficient of lift;
D_{ij} = grain diameter of grain size fraction i of mineral density j (m);
D_{nij} = nominal grain diameter of grain size fraction i of mineral density j (m);
D_* = dimensionless grain diameter;
D_{50} = median grain diameter of active layer (m);
d = flow depth (m);
e = smallest instantaneous shear stress interval greater than τ_{cij};
F_{ij} = volumetric proportion of grain size interval i and mineral density j in active layer;
f = largest instantaneous shear stress interval less than suspension shear stress;
g = gravitational acceleration (m/s^2);
h = constant in (12) and (30);
i_b = bedload transport rate in immersed weight per unit width per unit time (kg/s^3);
i_s = depth-integrated suspended load transport rate per unit width (kg/s/m);
k_M = coefficient in Manning's equation ($1\ m^{1/2}s^{-1}$);
k_s = characteristic bed roughness (m);
k_{1-10} = parameters in James entrainment function;
m = coefficient in Komar entrainment function;
n = Manning's roughness coefficient ($m^{1/6}$);
nd = node number;
P_k = proportion of time kth shear stress interval is active;
p = bed porosity;
Q = flow discharge (m^3/s);
R = hydraulic radius (m);
Re_D = grain Reynolds number;
S_f = friction slope;
S_0 = bed slope;
T_a = active layer thickness (m);
t = time (s);
U_{bij} = near-bed velocity of grain size i of mineral density j (m/s);
u_{*cij} = critical shear velocity necessary to entrain grain size i of mineral density j (m/s);
u'_* = skin friction component of mean, time-averaged bed shear velocity (m/s);
V_x, V_y, V_z = fluid velocity along the x-, y-, and z-axis, respectively (m/s);

$V_x(z)$ = longitudinal velocity as function of depth (m/s);
$\bar{V}_x$ = mean longitudinal flow velocity (m/s);
W_* = dimensionless settling velocity;
w_{ij} = settling velocity of grain size interval i of mineral density j (m/s);
x = distance along axis parallel to bed in flow direction (m);
x' = vertical profile correction factor in James entrainment function;
y = distance along axis parallel to bed in cross-flow direction (m);
z = distance normal to bed (m);
z_{bij} = bed elevation attributable to ith grain size fraction of mineral density j (m);
$\tan \alpha$ = dynamic friction coefficient in modified Bagnold equation;
β = bed slope angle;
ε_s = sediment eddy diffusion coefficient in vertical (m²/s);
ε_z = fluid eddy diffusion coefficient in vertical (m²/s);
Θ_{c50} = dimensionless critical shear stress necessary to entrain median grain diameter;
Θ_0 = dimensionless bed shear stress;
Θ_{cij} = dimensionless critical shear stress necessary to entrain grain i of mineral density j;
κ = von Karman's constant;
ν = kinematic fluid viscosity;
ρ = fluid density (kg/m³);
σ_j = mineral density j (kg/m³);
σ_τ = standard deviation of shear stress distribution (Pa);
τ' = instantaneous effective bed shear stress (Pa);
τ_{cij} = critical bed shear stress necessary to entrain grain size i of mineral density j (Pa);
τ_{c50} = critical bed shear stress necessary to entrain median grain diameter (Pa);
$\bar{\tau}'$ = skin friction component of time-averaged, median bed shear stress (Pa);
Φ = weighting factor for Preissman difference scheme; and
ϕ = grain pivot angle.

Subscripts and Superscripts

a = at height a above bed;
b = bedload;
i = grain size interval;
j = mineral density;
k = shear stress interval number;
nd = node number;
s = suspended load;
z = at height z above bed; and
$'$ = skin friction component.

Routing of Heterogeneous Sediments over Movable Bed: Model Verification

By Koen R. Vogel,[1] Andre van Niekerk,[2] Rudy L. Slingerland,[3] and John S. Bridge[4]

Abstract: A one-dimensional numerical model of heterogeneous size-density sediment transport has been developed to simulate the movement of graded sediments in natural and laboratory flow reaches. Predicted temporal and spatial variations in bed and armor-layer grain size distributions, eroded grain size distribution, eroded thicknesses, and total bedload transport rates compare quite favorably to observed variations in flumes, the San Luis canal, Colorado, and the East Fork River, Wyoming, for a large variety of flow scales and flow conditions. The main advantages of this model over others is the high degree of accuracy of model results obtained using only bed and flow variables as input, the treatment of turbulent fluctuations of bed shear stress, the minimization of calibration factors, and the explicit treatment of multiple grain densities. In addition, only the active layer thickness must be calibrated.

Introduction

A model investigating density and size sorting (MIDAS) has been developed (van Niekerk et al. 1992) to predict the transport of heterogeneous size-density sediments under nonuniform, quasi-unsteady, cross-sectionally averaged flow conditions. MIDAS was developed to accurately predict temporal and spatial variations in: (1) The flow field; (2) the size-density distributions of the bed material; (3) total transport rates of all size-density fractions making up the bed; and (4) bed degradation and aggradation.

The purpose of this article is to demonstrate that MIDAS performs well when compared against flume data of Little and Mayer (1972) and Ashida and Michiue (1971), and field data from the San Luis Valley canals collected by Lane and Carlson (1953) and from the East Fork River data collected by Leopold and Emmett (1976) and Mahoney et al. (1976). In addition, its use in predicting the occurrence of heavy mineral placers in a generic alluvial fan is demonstrated.

Specific Formulation of Model

The input parameters used in the model (van Niekerk et al. 1992) can be divided into three categories: (1) Physical constants known with some precision, e.g., the gravitational acceleration, dynamic viscosity of fluid, etc.; (2) numerical parameters used in discretization and solution schemes, e.g., the number of distance steps or the number of shear stress intervals; and (3) parameters defining how the model approaches the physics of the prob-

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