Necessary conditions for a meandering-river avulsion

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ABSTRACT

There is as yet no rational basis for predicting the conditions leading to avulsion of a meandering river. Here we present a conceptual model and quantify it under simplifying assumptions as a first step toward the construction of a stability diagram for avulsion initiation. It is assumed initially that a rectangular crevasse channel of arbitrary depth is cut into the levee of a meandering river. Because the water entering the crevasse channel is derived from relatively high in the main flow, it contains low concentrations of suspended solids. Consequently, the crevasse flow is under capacity and the entrance is eroded. Deepening of the entrance increases the crevasse-channel discharge, and the concentration of suspended solids supplied, because more sediment-laden waters are tapped from the deeper flow in the main channel. The crevasse entrance is predicted to deepen until its sediment-carrying capacity is satisfied by the suspended solids entering from the main channel. Whether an avulsion will occur for a particular combination of initial conditions depends upon whether there exists steady-state hydraulic and sediment-transport conditions for crevasse channel depths that are equal to or less than the main-channel depth. This conceptual model is quantified by writing the unsteady, gradually varied, one-dimensional conservation of mass and momentum equations for water and sediment transport through a main and crevasse channel. Sediment transport is computed as the integral of a cross-sectional mean velocity and a Rouse concentration profile. Solutions show that stable crevasse channels exist only for particular combinations of the initial height of the crevasse bed relative to the water depth in the main channel and the ratio of initial crevasse bed slope to main-channel slope.

INTRODUCTION

Avulsions, wherein a channel belt is abruptly abandoned for a new course at a lower elevation, are known through modern examples such as the Mississippi (Fisk, 1947; Saucier, 1994; Gomez et al., 1995), the Meander in Anatolia (Russell, 1954), the Kosi in India (Gole and Chitale, 1966; Wells and Dorr, 1987), the Saskatchewan in Canada (Smith et al., 1989; Smith and Pérez-Arlucea, 1994; Smith et al., 1998), the Thomson in Australia (Brizga and Finlayson, 1990), the Okavango in Africa (McCarthy et al., 1992), and the Owens and King Rivers in Australia (Schumm et al., 1996) as well as through interpretations of the geologic record (Aslan and Kraus, 1993; Törnqvist, 1994; Willis and Behrensmeyer, 1994; Kraus, 1996; and Jorgensen and Fielding, 1996, to name only a few recent studies). Given the impact of avulsions on society and their role in creating alluvial strata, it is surprising that so little is known about their origin. We still cannot predict the necessary and sufficient conditions that give rise to an avulsion, nor the evolution of the subsequent channel network.

The present theory of the avulsion process on a meandering river starts with a relatively long-lived meander belt within a flood plain. Sedimentation rates are known to be higher in the meander belt than in the flood basin (Allen, 1965; Bridge and Leeder, 1979; Sadler, 1981; Pizzuto, 1987; James, 1985; Bryant et al., 1995). Through time, these differential rates create an alluvial ridge. An avulsion is thought to occur when the cross-valley slope reaches some (unspecified) multiple of the down-channel slope, allowing flood waters to irreversibly enlarge a preexisting crevasse or random topographic low in the levee. This standard model has been translated into a heuristic model by Mackey and Bridge (1995), who give the probability of an avulsion occurring at a specific site as the product of the slope ratio times the return period of flood discharge in excess of some threshold value. Implicit in Mackey and Bridge's formulation is the idea that avulsion frequency is more strongly controlled by sedimentation rate than by the absolute amount of sedimentation, a point elaborated by Bryant et al. (1995) and Heller and Paola (1996). Although intuitively reasonable, the standard theory seems never to have been tested.

The purpose of this paper is to present a physically based conceptual model of the avulsion process and then to use the model to predict the necessary hydraulic and geomorphic conditions for an avulsion to occur. Our results show that whether a crevasse of arbitrary initial geometry will trigger an avulsion depends upon sediment grain size, the initial depth of the crevasse, and the ratio of crevasse to main channel bed slopes.

CONCEPTUAL MODEL

Consider a reach of a main channel containing a crevasse channel of arbitrary cross section and bed slope cut into a levee such that the crevasse-throat bottom is at an arbitrary height l, above the main-channel bottom (Fig. 1). Whether the crevasse channel enlarges, heals, or remains unchanged will depend upon the amount of sediment delivered to its throat relative to its carrying capacity. During flood stage of the main channel (here defined as equal to the local levee height), the bed elevation of a crevasse throat is initially high up on the side of the main channel and taps relatively clear water from the upper waters. Therefore, for its discharge, the crevasse might be under capacity, causing erosion and increase in cross-sectional area. This erosion will continue until one of two conditions is met: (1) sediment flux to the throat from the main channel comes into balance with crevasse-channel capacity or (2) crevasse capacity is reduced by a decreasing water surface slope (not the crevasse bed slope), either because the flood basin fills with water or the crevasse channel elongates.

It is reasonable to think that condition 1 could be met, because suspended sediment concentration increases exponentially with depth in the main channel if such a load exists (C_s in Fig. 1B). Therefore, as the bottom of the crevasse channel lowers, sediment flux to the throat will increase. Whether condition 2 is met depends upon the hydraulic conveyance of the flood basin and, in particular, the relative rates of increase of water and sedi-

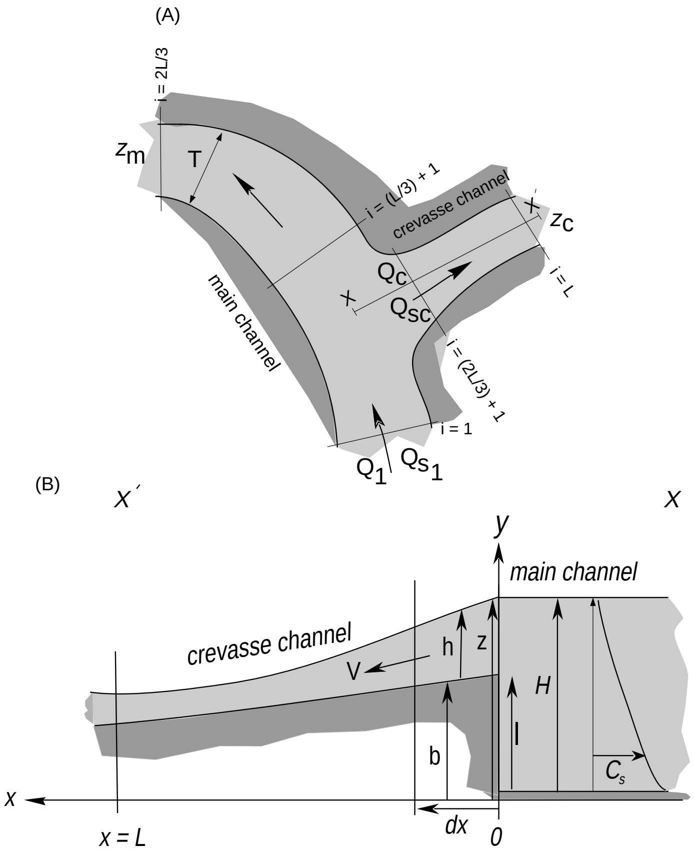

Figure 1. Conceptual model. A: Plan view. B: Cross section. L = maximum number of sections.

ment discharges down the crevasse. Whether and under what hydraulic conditions an avulsion will occur then reduces to finding the equilibrium solutions to the equations describing flow and sediment transport through the crevasse. Solutions in which the crevasse channel monotonically deepens to take all the main-channel discharge are called avulsions.

Exploring these solutions will not provide the complete answer, however, because floods are of limited duration, whereas crevasse enlargement proceeds at a finite rate. In the few reported cases such as the Poydras crevasse of 1922 below New Orleans (Barry, 1997) and the upper Mississippi crevasses of 1993 (Gomez et al., 1995), the crevasse throats did not deepen beyond half the main-channel depth before the flood waters abated. During the following normal river stage, the crevasse channel may still pass water, but its cross-sectional area will likely be too large for its discharge. Through-flowing waters will deposit side-channel bars, and suspended sediment advected transversely from the main-channel flow into the crevasse throat will form a bar, thereby partially filling the crevasse. At the next flood, the cycle of erosion and subsequent partial healing is repeated. This line of reasoning suggests that there may be a minimum depth of throat scour during a flood that will lead to an avulsion. Scour greater than this minimum allows sufficient flow through the crevasse during average stage to keep the throat clear of advected sediment. Crevasse enlargement can then be cumulative and, with time, will capture the total discharge of the main channel. This analysis also does not consider bedload, nor does it consider the potential sediment transport effects of flow acceleration in the crevasse. Finally, it is assumed that bed elevation in the main channel does not change with time. This assumption would be true (1) where the discharge down the crevasse initially is small compared to the discharge of the main channel or (2) when the crevasse enlarges rapidly.

TABLE 1. VALUES USED IN THESE EXPERIMENTS

Variable or constant	Value
Darcy-Weisbach friction factor, f	0.03
Dynamic friction coefficient, tan α	0.5
Channel width, T	100 m
Water discharge at head of main channel, Q_1	1000 $m^3 \cdot s^{-1}$
Constant in Rouse number, k	0.11

MATHEMATICAL MODEL

As an initial attempt to define the conditions necessary for an avulsion, temporally evolving bed elevations at points along the crevasse are calculated for selected combinations of hydraulic and sediment properties of the crevasse and main channel. The simplified geometry of the model is given in Figure 1. Assume that the main channel and crevasse channel are divided into a number of arbitrary cross sections i, where i = 1 to L, and each cross section is rectangular and of invariant width $T_{(i)}$ (m). The latter is justified for crevasse channels with well-developed levees. The dependent variables of the problem are the vertically averaged water velocity $V_{(i)}$ ($m \cdot s^{-1}$), water depth $h_{(i)}$ (m), and bed elevation $b_{(i)}$ (m), all functions of time and distance along the two channels. The three dependent variables can be computed from the one-dimensional differential equations describing gradually varied, unsteady flow over a movable bed:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} QV + gA\frac{\partial h}{\partial x} + \frac{fTV^2}{8} + gA\frac{\partial b}{\partial x} = 0 \quad (1)$$

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (2)$$

$$\frac{\partial b}{\partial t} + \frac{1}{pT}\frac{\partial}{\partial x}\left(Q_s\right) = 0 \quad (3)$$

$$Q_s = VT\int_{y_r}^{h} C dy = VT\int_{y_r}^{h} C_r \exp\left[\frac{a}{h}\left(y - y_r\right)\right] dy = \frac{QC_r}{a}\left(e^a - 1\right) \quad (4)$$

$$C_r = \frac{\tau_o - \tau_c}{\left(\tan\alpha\right) 2dg\left(\sigma - \rho\right)} \quad (5)$$

where Q = water discharge ($m^3 \cdot s^{-1}$); t = time (s); x = distance along channel (m); g = gravitational acceleration ($m \cdot s^{-2}$); A = cross-sectional area = Th (m^2); f = Darcy-Weisbach friction factor; p = volume of sediment in a unit volume of bed layer; Q_s = sediment discharge ($m^3 \cdot s^{-1}$), here taken to be solely suspended load; y_r = level at which reference concentration is measured, here taken to be the top of the moving bed layer and therefore $y_r/h \approx 0$; C = suspended-sediment concentration at height y; C_r = reference concentration for the suspended load; a = Rouse number, equal to $-w/(kU_*)$, where k = 0.1 for grain sizes of about 0.1 mm, w = fall velocity ($m \cdot s^{-1}$), and U_* = bed shear velocity ($m \cdot s^{-1}$); τ_o = fluid shear stress on the bed; τ_c = critical shear stress for the grain size d; tan α = dynamic friction coefficient (here set equal to 1/2; see Bridge and Bennett, 1992); and σ = grain density ($kg \cdot m^{-3}$) and ρ = fluid density ($kg \cdot m^{-3}$).

Equations 1and 2 are the St. Venant shallow-water equations and Equation 3 is the Exner equation describing sediment conservation. Equation 4 computes the suspended load by using McTigue's (1981) suspended-sediment concentration profile. The reference concentration is defined by Equation 5 as the concentration of grains of diameter d in the moving bed layer calculated by the modified Bagnold bedload transport formula (Bridge and Bennett, 1992).

Boundary and initial conditions are needed to complete the equation set. Boundary conditions consist of temporally invariant water (Q_1) and sediment (Q_{s1}) discharges at the upstream end of the main channel (i = 1 in Fig. 1), water-surface elevations at the downstream ends of the crevasse (z_c)

and main channel (z_m), and interior boundary conditions at the crevasse throat, where: $Q_{(L/3)+1} + Q_{(2L/3)+1}$ is set equal to Q_1. Especially important is the interior boundary condition for sediment delivery to the crevasse throat. As noted in the conceptual model above, sediment entering the throat is assumed to originate from the layer of water in the main channel that lies above the throat bottom. The mean concentration of sediment in that layer is given by

$$\overline{C} = \frac{1}{H-l}\int_l^H C_r \exp\left[\frac{a}{H}\left(y-y_r\right)\right]dy = \frac{HC_r}{a\left(H-l\right)}\left(e^a - e^{a\frac{l}{H}}\right), \qquad (6)$$

where C_r = reference concentration in the main channel immediately upstream of the crevasse, and H and l are defined in Figure 1. Then the volumetric flux of sediment entering the crevasse is given by

$$Q_{sc} = \left(Q\right)_{\left[\left(\frac{2L}{3}\right)+1\right]}\frac{HC_r}{a\left(H-l\right)}\left(e^a - e^{a\frac{l}{H}}\right). \qquad (7)$$

Initial conditions consist of arbitrary specifications of $Q_{(i)}$, $V_{(i)}$, and $z_{(i)}$, and a particular bed geometry specified by $b_{(i)}$. The initial flow variables can be arbitrary because the hydraulic part of the model is first computed to steady-state conditions before the bed is loosened. Uncoupled, unsteady solutions are obtained by first solving Equations 1 and 2 at each whole time step using the unconditionally stable, linear, implicit, finite-difference method of Chen (1979). Once the bed is loosened, Equations 3, 4, 5, and 7 are then solved at each half time step. Computation continues until either the crevasse takes all the water discharge, heals, or reaches steady state. Table 1 lists various boundary and initial values used in these experiments.

RESULTS

Results of the numerical experiments are plotted in Figures 2 and 3 for grain-settling velocities of 0.01 and 0.06 m/s (0.1 and 0.4 mm grain diameters, respectively). The ratio of initial crevasse to main-channel bed slope was varied from 0.7 to 10, which by design caused proportional variations in water-surface slopes. Also varied was the initial bed elevation of the crevasse throat, here called the lip height, and measured as the height of the crevasse throat bottom relative to the main-channel depth. Lip heights varied from 0.2 to an upper limit of 0.7, above which flow in the crevasse became supercritical and incomputable by this finite-difference scheme.

For both grain sizes there exists an upper limit of relative crevasse bed slope above which avulsion ultimately occurs. This limit is dependent upon relative lip height, although only weakly so for the coarser grain size. Initial conditions above this boundary always resulted in avulsion; below this boundary, solutions obtained equilibrium in one of two states: run-away alluviation, here called "healing" of the crevasse, or a steady-state crevasse flow in which the sediment and water flux entering the main channel were partitioned between the crevasse and downstream main channel such that both were at grade.

This general behavior arises because the crevasse lip height controls both the sediment-transport rate at capacity down the crevasse and the flux of sediment and water entering the crevasse from the main channel. Consider the case in which the initial sediment feed to the crevasse is greater than the sediment-carrying capacity of the crevasse flow. In this case, the crevasse lip increases in elevation owing to local sedimentation. This process increases the local slope down the crevasse, increasing the crevasse's sediment-transport capacity because the steady-state sediment flux as derived from Equation 4 is

$$Q_s \propto \left(H - l\right)^{\frac{5}{2}} S^{\frac{3}{2}} \qquad (8)$$

if variation in the Rouse number is small compared to variations in Q and S. But increasing lip height *decreases* the water discharge in the channel [represented by $(H - l)$], and because the exponent on $(H - l)$ is greater than the exponent on S, the result is a net decrease in Q_s of the crevasse flow. By this reasoning, it would seem that even the slightest sedimentation in the throat would result in healing. But increasing the lip elevation also decreases the amount of sediment entering the crevasse because, by Equation 7,

$$Q_{sc} \propto \sqrt{\left(H - l\right)S}\left(1 - e^{a\frac{(H-l)}{H}}\right). \qquad (9)$$

Whether the crevasse alluviates, enlarges, or reaches an equilibrium depends upon whether a balance between these two sediment fluxes can be reached, starting from the initial and boundary conditions of the system.

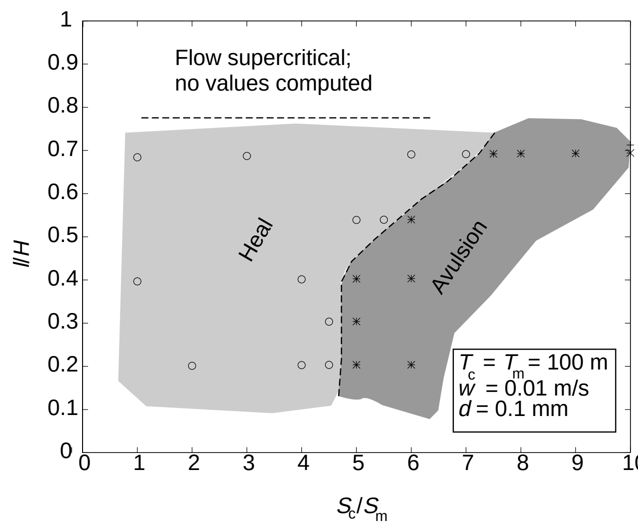

Figure 2. Predicted behavior of crevasse channel carrying very fine sand as a function of initial relative lip height and relative bed slope. Circles represent starting conditions of numerical experiments for which crevasse sedimented shut. Asterisks represent starting conditions for which crevasse channel captured total discharge. *S* = bed slope; *T* = channel width; c = crevasse; m = main channel; *w* = grain settling velocity; *d* = grain diameter; see Figure 1 for other variable definitions.

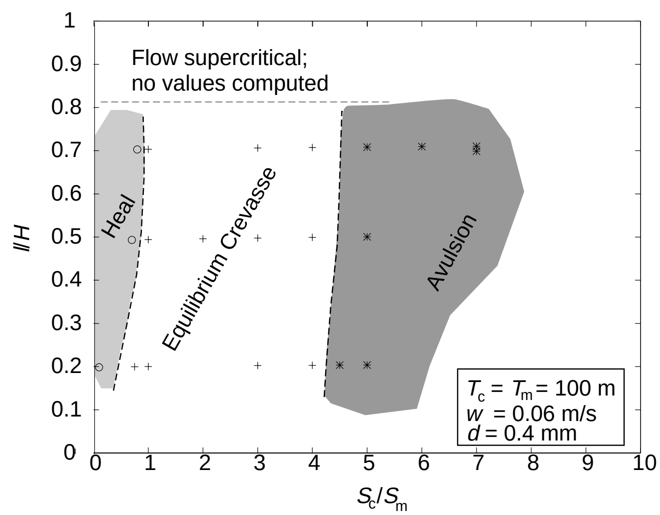

Figure 3. Predicted behavior of crevasse channel as a function of initial relative lip height and bed slope for medium sand. Meaning of circles and asterisks same as in Figure 2. Pluses represent starting conditions of numerical experiments for which crevasse channel passed a part of the total discharge at steady state. See Figure 2 for variable definitions.

It is interesting that there is a domain of equilibrium crevasses for the coarser grain size (cf. Figs. 2 and 3) but not for the finer. We think that this behavior arises because coarser grain sizes are distributed less uniformly through the water column. Therefore, for coarser sizes, a unit change in lip elevation of the crevasse throat produces a greater change in sediment feed to the crevasse, allowing the sediment feed to come into balance with the capacity of the crevasse.

FIELD DATA

These predictions can be compared to the hydraulic and geomorphological relationships of the Saskatchewan River near Cumberland House, Saskatchewan, where an avulsion occurred in the 1870s. Mean annual discharge from historical records is 500 ($m^3 \cdot s^{-1}$) and bankfull discharge is about 2800 ($m^3 \cdot s^{-1}$) (PFRA, 1954, 1955). Mean grain size of the suspended load is between 0.1 and 0.2 mm (Carson, 1990). As documented in Smith et al. (1998), a preexisting tributary or crevasse channel was the site of the avulsion. We do not know the preavulsion lip height. We do know, however, that levee slopes on the preavulsion channel average about 0.001 or 10 times the regional downvalley slope. If the preavulsion slopes were similar, then the 1870s avulsion is consistent with model predictions in Figure 2.

Farther downstream along the new avulsion network of the Saskatchewan, smaller crevasses have opened, extended into fen meadows, and healed within a few decades. Again, we do not know the relative lip height when the crevasses were active, but we can estimate the relative slopes at the time of healing by measuring present crevasse-channel slopes. Relative slopes range from 3 to 5, which, for a grain size of 0.1 mm, plot in the appropriate part of Figure 2.

CONCLUSIONS

A simple one-dimensional model of bifurcating flow in a single-thread river shows that whether a crevasse heals, runs away to an avulsion, or reaches a steady state depends upon the ratio of crevasse to main-channel bed slopes, the height of the crevasse bottom above the bed of the main channel, and bed grain size. For fine to medium sand, crevasse slopes greater than about eight times the main-channel slope are predicted to capture all the main flow, regardless of initial crevasse size. These results do not account for variations in flow discharge, nor do they address the conditions necessary for the initial creation of the crevasse channel.

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