ROLE OF HYDRAULIC SORTING IN THE ORIGIN OF FLUVIAL PLACERS[1]

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ABSTRACT: A necessary condition for placer development is selective sorting at the grain scale by size and density. The effects of differential entrainment, suspension, and transport on an initial distribution composed of medium-size quartz and 10% fine-size magnetite were modeled by solving the Einstein bedload function for specific grain friction velocities ($U^{*\prime}$) in the range 3–63 $cm \cdot sec^{-1}$ and bed roughnesses of 0.55, 2, 5, and 10 mm. For any value of $U^{*\prime}$ and for both mineral densities, the transport rate for all sizes in the initial distribution decreases with increasing roughness, the decrease being greatest in the finer sizes. The concentration of magnetite transported in the flow increases with increasing $U^{*\prime}$ for each roughness and decreases with increasing roughness for each $U^{*\prime}$. The settling velocity ratio of magnetite to quartz in transport decreases with increasing $U^{*\prime}$ for any value of roughness. For any value of $U^{*\prime}$, the ratio first decreases then increases with increasing roughness, ranging from 1.36 at low $U^{*\prime}$ and roughness to 0.76 at high $U^{*\prime}$ and intermediate roughness. These results are due to variations in the reactive angles of grains and the extent to which grains hide in the lower, inner zone of the boundary layer. Concentrations of heavy minerals at the bed, bar, and system scales are explained using these results.

INTRODUCTION

In the summer of 1896 George Carmack announced to the miners in the Caribou Saloon at Fortymile, Yukon Territory, that he had discovered gold on a tributary of the Tron-diuck River. No one was much impressed though; Carmack had a reputation for tall tales and short accomplishments. He poured some flakes out on the bar. It was true, no miner had seen that shape of gold before. By sheer luck, George Carmack had discovered Bonanza Creek, the richest gold placer in the Klondike and possibly in the world (McPhee 1976).

Unfortunately, even today luck plays a considerable role in placer exploration because the conditions producing this type of heavy mineral enrichment remain imperfectly known. Numerous studies, cited later, have made it clear that the important local variables are: 1) the settling velocity distributions of the local populations of heavy and light minerals, 2) the long-term flow hydraulics of the site, 3) the average roughness size of the bed, and 4) the volume of material processed through time. Less clearly understood are the enrichment processes themselves at the grain level and the optimal combinations of the above variables that control these processes.

How does grain-by-grain selective sorting by density actually occur? Although it seems obvious that under a given flow, the heavy minerals will have a lower probability of being entrained, the same seems true for larger less-dense grains. If the heavy minerals are generally smaller in mean size in the local source, as is commonly the case, then the concentration of the lag need not necessarily rise above background.

At a larger scale, what combinations of variables have caused paleoplacers to develop at different sites within seemingly similar paleodepositional systems? For example, McGowen and Groat (1971) discuss an alluvial fan complex from west Texas in which the heavy mineral concentrations occur in distal portions, whereas Minter (1978) discusses a Witwatersrand alluvial fan complex in which heavy minerals are concentrated in proximal to medial portions.

The problem is complicated, but following the lead of Brush (1965), some progress can be made by analysing the influences four local hydraulic processes have on an initial population of light and heavy minerals. The approach used here is to calculate the size ranges

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TABLE 1.—*Observed sites of heavy mineral placers in the fluvial system*

System Scale (10^4 m)	
Bands parallel to depositional strike	Smith and Minter (1980); McGowen and Groat (1971).
Heads of wet alluvial fans	Schumm (1977).
Points of abrupt valley widening	Kuzvart and Bohmer (1978); Crampton (1937).
Points of exit of highland rivers onto a plain	Toh (1978).
Regional angular unconformities	Minter (1978).
Bar Scale (10^2 m)	
Concave sides of sharp bends	Kuzvart and Bohmer (1978).
Convex banks of meanders	Kuzvart and Bohmer (1978).
Heads of mid-channel bars	Toh (1978); Smith and Minter (1980); Kartashov (1971).
Point bars with suction eddies	Toh (1978); Bateman (1950).
Scour holes, esp. at tributary confluences	Kuzvart and Bohmer (1978); Mosley and Schumm (1977).
Inner bedrock channels and false bedrock	Schumm (1977); Kuzvart and Bohmer (1978); Adams et al. (1978).
Bedrock riffles	Cheney and Patton; (1967); Toh (1978).
Bed Scale (10^0 m)	
Scoured bases of trough cross-strata sets	Toh (1978); McGowen and Groat (1971).
Winnowed tops of gravel bars	Toh (1978); McGowen and Groat (1971).
Tangential toes of foresets	Toh (1978); McGowen and Groat (1971); Smith and Minter (1980).
Thin ripple-form accumulations on dune stoss slopes	Brady and Jobson (1973).
Dune crests	Brady and Jobson (1973).
Dune foreset beds	Brady and Jobson (1973); McGowen and Groat (1971).
Plane parallel laminae	Slingerland (1977).
Leeward side of obstacles	Lindgren (1911).

and abundances of light and heavy minerals from a population that are: 1) entrained off a bed together, 2) suspended at the same elevation in a turbulent, open channel flow, 3) moved to the same level in a concentrated granular dispersion, and 4) transported at the same weight ratio. The sorting that occurs from these processes here is called respectively, *entrainment sorting, suspension sorting, shear sorting,* and *transport sorting.* The results of this analysis explain, at least qualitatively, the occurrences, settling velocities, and grades of some typical heavy mineral placers in streams.

OCCURRENCES AND CHARACTERISTICS OF HEAVY MINERAL DEPOSITS IN STREAMS

Abundant field observations of fluvial placers have shown that concentrations of heavy minerals occur at preferred sites and at different scales. These sites are tabulated and arranged in Table 1 following a classification modified from Smith and Minter (1980).

The concentrations and settling velocity relationships of light and heavy minerals in placers and in sedimentary deposits generally, have been studied by Rittenhouse (1943), Van Andel (1950), Sundborg (1956), McIntyre (1959), Hand (1967), Briggs (1965), White and Williams (1967), Grigg and Rathbun (1969), Lowright et al. (1972), Stapor (1973), Slingerland (1977; 1980), and Sallenger (1979). The derived relations between heavy mineral abundance and settling-velocity ratio separate into two groups: one in which the roughness for a deposit is determined by grains in transport, and one in which a preexisting coarse substrate is trapping a moving population. In the first group, sand deposits sampled at the lamination to thin-bed scale show a heavy mineral abundance decreasing with increasing sample mean settling velocity because under common geological conditions larger heavy mineral grains are simply not as abundant as larger light mineral grains (compare Osovetskii 1974). Exceptions occur close to the source and are of special economic interest because they are potential placers. Also in this group, the heavy mineral abundance increases with increasing heavy to light settling velocity ratio. Flows commonly enrich deposits by entraining or depositing size fractions of the original sediment such that the heavy minerals become more nearly the same size or larger than the lights, thus increasing the heavy to light settling velocity ratio.

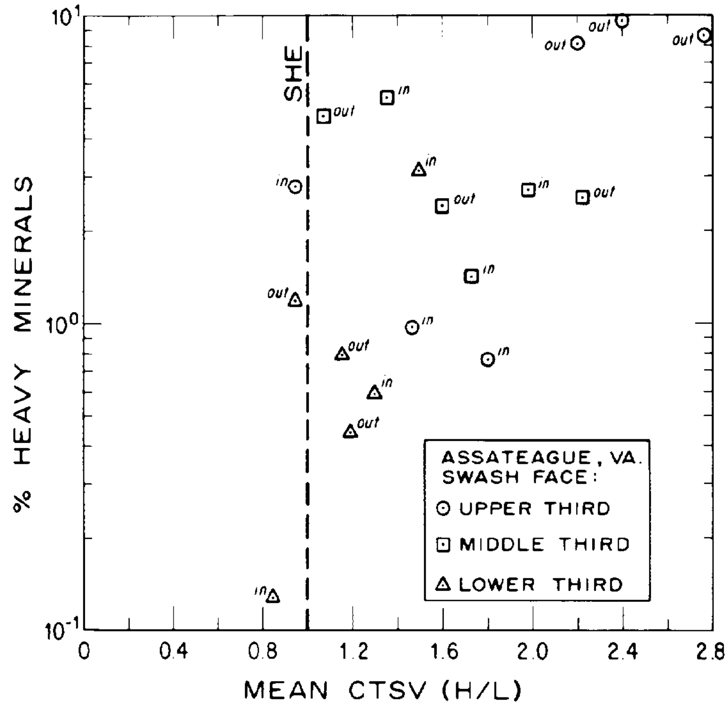

FIG. 1.—Ratios of mean constant terminal settling velocities (CTSV) of ilmenite (H) and quartz (L) and percentage of heavy minerals in surface samples on a swashface. Samples marked "in" were taken at the time of maximum swash advance, and samples marked "out" were taken at maximum swash retreat. SHE = settling hydraulic equivalence.

For example, consider three locations on the surface of a swashface on Assateague Beach, Virginia, from the plunge point to the swash top. Here, the processes of transport, deposition, and reentrainment occur over each swash advance and retreat, thereby simulating the longer-term flow variability in streams. The swashface is about 10 m long, dips 4°, has a mean quartz size of 0.4 mm, and contains a range of heavy mineral sizes in the finer fractions. Simultaneous samples of the upper several grain layers taken at the time of maximum swash advance (Fig. 1, "in") and maximum retreat (Fig. 1, "out") show a general trend of increasing heavy mineral percent with increasing settling velocity ratio. Samples from the three locations plot in groups: all plunge-point samples on both the advance (in) and retreat (out) swashes plot near the equal settling line with no difference in heavy mineral percentages. Upper swash samples show two groups: The "ins" plot closer to the equal settling line and are less heavy mineral rich than the "outs." These observations are explained later to be the result of alternately settling grains and preferentially reentraining larger, lighter grains.

In the second group, the characteristics of the trapped deposits are not well known. Flume experiments by Minter and Toens (1970) showed that heavy mineral concentrations are greater in thinner layers of gravel but proportional to and slightly less than their concentrations in the transported bedload. Theoretical results, presented later in this paper, show this to be true only if the median settling velocity of the light minerals is many times that of the heavy minerals.

TYPES OF SELECTIVE SORTING

To explain the occurrences and characteristics described above requires as a necessary condition a sorting process at the grain scale. Four are discussed: entrainment, suspension, shear, and transport sorting. The most important of these is transport sorting because it subsumes entrainment and suspension sorting, and it will be emphasized here.

Entrainment Sorting.—Entrainment sorting is the separation of grains into distinct populations of different size, density, and shape by differential pick-up off a bed. Of the four types of sorting, it has received the most attention (Sundborg 1956; Brady and Jobson 1973; Grigg and Rathbun 1969; Ljunggren and Sundborg 1968; Saks and Gavshina 1976; Slingerland 1977) even though it only explains the characteristics of lag deposits. The usual procedure is to calculate the critical velocity or shear stress needed to obtain a fluid force greater than a resisting force in a torque balance on one grain. The important variables are the friction velocity (U^*), grain diameter (d_{mm}), grain density (ρ_p), and bottom roughness size (k).

Results show that for grains defining their own roughness, the U^*_c versus d_{mm} curve rises monotonically for any one density. Curves for different densities are similar in shape but displaced at different U^*_c levels (compare Ljunggren and Sundborg 1968). But for an initial fixed roughness, the U^*_c versus d_{mm} curves for any one density are higher order polynomials (compare Slingerland 1977). With increasing grain size, U^*_c is high for grains much smaller than k due to sheltering effects, falls to a minimum for grains somewhat larger than k, and then rises again for much larger grains.

As an example of entrainment sorting consider a deposit 10% magnetite by volume whose distribution is given in Figure 2. It consists of arbitrary phi-normal distributions of quartz and magnetite with the magnetite smaller than the total distribution mean size which is 1.23 ϕ (0.43 mm). Experience suggests that commonly in sands the magnetite mean size is near 2.15 ϕ (0.2 mm), that is, a little smaller than the settling-equivalent quartz size. The magnetite distribution is usually better sorted. If this population comes to rest on a substrate whose size is nearer in size to the smaller heavy grains, the larger lighter grains will be more susceptible to entrainment because they protrude higher into the boundary layer and have smaller reactive angles (Slingerland 1977). Any lag deposit formed by subsequent flows will then consist of a heavily enriched lamination whose heavy to light settling velocity ratio is greater than 1. If the roughness size is nearer in scale to the larger lighter grains, the intermediate-sized heavy minerals, even though denser, will be entrained along with finer light grains, as turbulent vortices pluck grains from among roughness elements. The resulting deposit will have a settling velocity ratio less than 1 and will not be as enriched.

Suspension Sorting.—Suspension sorting is the fractionation of grains of different settling velocities into different levels off the bed in a turbulent, open-channel flow, and their subsequent separation into different deposits. Brush (1965) discussed its role in progressive downstream sorting. The sizes of light and heavy minerals that occur together at any one location in the flow may be calculated from the Rouse equation (1950):

$$\frac{c}{c_a} = \left(\frac{D-y}{y}\cdot\frac{a}{D-a}\right)^{z_0}, \qquad (1)$$

where $z_0 = w_0/U^*k_0$, D = water depth, y = elevation off the bed, a = elevation of reference concentration, c_a = reference concentration, c = concentration of suspended load, w_0 = constant terminal settling velocity, U^* = friction velocity, and k_0 = Von Karman constant.

For any elevation in the flow, the relationship between relative dimensionless concentration and relative settling velocities for heavy and light grains is:

$$\left(\frac{c}{c_a}\right)_h = \left(\frac{c}{c_a}\right)_l^{w_h/w_l}, \qquad (2)$$

where w_h and w_l are the settling velocities of grains at the top of the moving bed layer. Saks (1974) and Tourtelot (1968) have emphasized the role of settling velocity in the formation of placers.

Consider again the distribution in Figure 2 that is 10% magnetite by volume. The mean settling velocity of the heavy grains is less than the mean settling velocity of the lighter grains in the bed, the ratio being 0.92. Then the relative concentration of heavy to light minerals suspended in the flow will be greater than in the moving bed layer at the reference location. If these grains were suspended in a turbulent open-channel flow, tapped off at

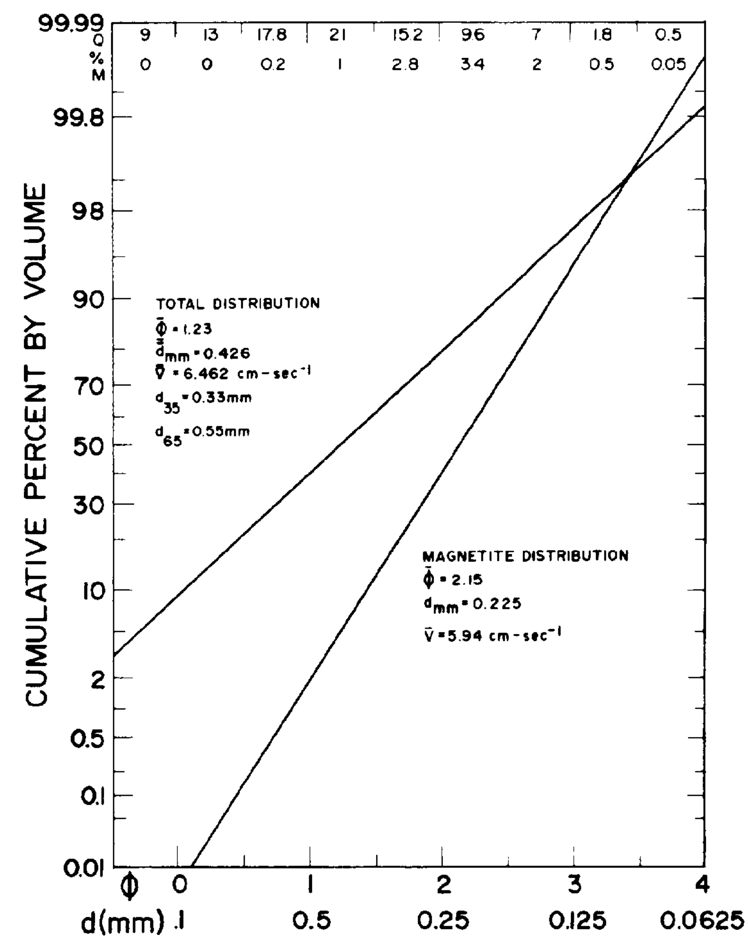

FIG. 2.—Initial ϕ-normal size distributions of 90% quartz and 10% magnetite. The percentages of quartz and magnetite in each size fraction that were entered into the bedload function are given at the top of the figure.

some elevation above the bed, and deposited, the deposit would be heavily enriched relative to the starting bed material.

Shear Sorting.—Shear sorting is the separation of grains into different horizons within a concentrated granular dispersion such as a moving bed layer or a grain flow. Its origin is due to the dispersive pressures arising from grain collisions (Bagnold 1954; Sallenger 1979) or kinetic sieving wherein smaller grains fall between larger ones (Middleton 1970).

In the first case, Bagnold (1954) showed that the dispersive pressure is proportional to particle size and density. If Bagnold's equation applies to nonuniform sizes, it predicts that larger or denser grains will be driven to a free surface. Two grains of different density coming to rest at the same horizon would have size ratios given by (Sallenger 1979):

$$\frac{d_h}{d_e} = \left(\frac{\rho_e}{\rho_h}\right)^{1/2}. \tag{3}$$

As an example of shear sorting by density, again consider the deposit 10% magnetite by volume whose size distribution is given in Figure 2. After ideal sorting by shearing, the deposit would have the vertical profiles of quartz size and magnetite concentration given in Figure 3. For these particular distri-

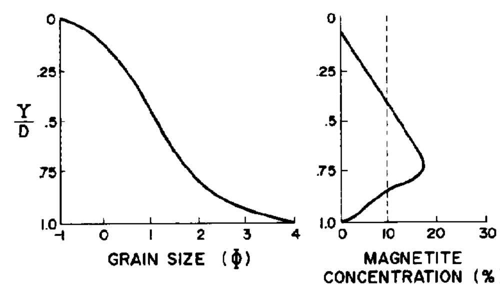

FIG. 3.—Quartz grain size and magnetite concentration through a sheared deposit whose original magnetite and quartz size distributions are given in Fig. 2 and whose original magnetite concentration was 10% (dashed line). In the shear-sorted deposit the magnetite becomes concentrated in the lower middle. Y/D = relative elevation; origin at the top of the deposit.

butions, the horizon from Y/D = 0.45 to 0.85 becomes enriched by up to twice the original concentration of magnetite. Shifting the original magnetite distribution towards the finer sizes would eliminate the heavy mineral impoverished base and produce profiles like those found on beach swash faces and washovers (Sallenger 1979).

Kinetic sieving, the second case, is the fractionation of grains in an agitated granular mass as smaller or denser grains fall downward between larger grains. The resulting deposit should coarsen and become less heavy mineral-rich upward. It is not presently possible however, to calculate the sizes of two different density grains that would come to rest together in any one horizon.

Transport Sorting.—Transport sorting is the fractionation of grains due to differential transport, by which is here meant, differences between the unit sediment transport rates of light and heavy minerals. These differences are due to variations in the probability of entrainment as well as in the motion and mean velocity of a grain already moving in the flow. Thus, transport selective sorting includes entrainment and suspension sorting.

The approach here is to use Einstein's bedload function to calculate the transport rates of light and heavy minerals under conditions of interest. Of all the known formulae, it best simulates the transport of denser mineral grains constituting a small percentage of the total distribution of sizes on the bed because it accounts for smaller grains hiding among larger grains. To do this, two factors are important: Y, a pressure correction factor, and ξ, a "hiding factor." Because of these, the bedload function predicts lesser values of transport for sediment moving over a much coarser substrate.

The limited experimental data available support these predictions. Gilbert (1914) showed for two mixed grades of quartz, $\bar{d}_{mm} = 0.304$ and $\bar{d}_{mm} = 4.94$, that the transport capacity of the finer fraction was reduced as the bed became made up of increasingly larger percentages of the coarser (Fig. 56, p. 174). Meland and Norrman (1966) showed in particle overpassing experiments that for the same U*, the velocities of particles of size 0.203 mm were an order of magnitude slower than particles of size d = 0.393 mm or d = 0.776 mm when the bed roughness was equal to 0.775 mm. Thus, the bedload function is, at the very least, qualitatively accurate in simulating the conditions of interest.

The computer program used here is modified from Shulits and Hill (1968) and follows the Einstein-Chow condensation of Einstein (1964). Einstein and Chien (1953) suggested a modification to the ξ function but in 1964 Einstein was apparently still satisfied with his first formulation.

Now, to provide a concrete example of transport sorting, consider a channel whose bed is comprised of the distribution in Figure 2, magnetite constituting 10% of the total distribution by volume. The channel is given the arbitrary dimensions of 30.9 m width and 0.41 m flow depth. Bank friction is not considered. What will be the total unit transport rates ($kg \cdot sec^{-1} \cdot meter\ width^{-1}$) of the quartz and magnetite at different energy slopes and bottom roughnesses? To answer this the computerized version of the bedload function was used with the sizes and fractions of the quartz and magnetite given in Figure 2. Quartz and magnetite transport rates were calculated in separate computer runs. For each of three different energy slopes, solutions were calculated for a bed roughness equal to d_{65} of the total distribution as well as for roughnesses of 2, 5, and 10 mm. Results are given in Table 2.

At a stream energy slope of 0.001, $U^{*\prime}$ = 3.14 $cm \cdot sec^{-1}$. $U^{*\prime}$ is that portion of the total

TABLE 2.

Magnetite Unit Rate of Transport ($kg \cdot sec^{-1} \cdot m^{-1}$)

		$U^{*\prime} = 3.14$ ($cm \cdot sec^{-1}$) d_{65} (mm)	$U^{*\prime} = 20.1$ ($cm \cdot sec^{-1}$) d_{65} (mm)				$U^{*\prime} = 63.8$ ($cm \cdot sec^{-1}$) d_{65} (mm)			
d(mm)	Per-cent	0.55	0.55	2.0	5.0	10.0	0.55	2.0	5.0	10.0
0.105	0.5	0	0.47 (8.6)	5.15×10^{-5} (612)			20.0 (3.7)	0.83 (5.1)	3.68×10^{3} (47.8)	0
0.149	2.0	0	1.41 (13.8)	2.06×10^{-3} (137)	0	0	108.3 (4.8)	4.50 (5.5)	6.05×10^{-2} (27.1)	0
0.210	3.4	0 (∞)	1.56 (16.1)	5.37×10^{-3} (97)	0	0	231.5 (4.8)	8.91 (5.1)	2.55×10^{-1} (15.3)	1.58×10^{-5} (4476)
0.300	2.8	1.40×10^{-4} (61)	0.81 (36.9)	6.12×10^{-3} (123)	0	0	200.0 (11.8)	7.53 (11.8)	3.95×10^{-1} (24.2)	8.51×10^{-4} (439)
0.420	1.0	1.29×10^{-4} (172)	0.20 (98.6)	5.10×10^{-3} (186)	0	0	44.4 (58.3)	2.55 (58.8)	1.27×10^{-1} (88.9)	2.0×10^{-3} (620)
0.590	0.2	2.24×10^{-5} (927)	0.03 (217)	2.78×10^{-3} (285)	0	0	4.35 (298)	0.49 (301)	3.86×10^{-2} (381)	1.06×10^{-3} (1629)
0.840	0	0†	0	0	0	0	0	0	0	0
1.19	0	0	0	0	0	0	0	0	0	0
		1.94×10^{-4}	4.47	2.15×10^{-2}			6.07×10^{2}	2.47×10^{1}	9.29×10^{-1}	3.92×10^{-3}

Quartz Unit Rate of Transport ($kg \cdot sec^{-1} \cdot m^{-1}$)

		$U^{*\prime} = 3.14$ (mm) d_{65} (mm)	$U^{*\prime} = 20.1$ ($cm \cdot sec^{-1}$) d_{65} (mm)				$U^{*\prime} = 63.8$ ($cm \cdot sec^{-1}$) d_{65} (mm)			
d(mm)	Per-cent	0.55	0.55	2.0	5.0	10.0	0.55	2.0	5.0	10.0
0.105	1.8	0	4.01	3.15×10^{-2}	0	0	7.38×10^{1}	4.28	1.76×10^{-1}	5.70×10^{-5}
0.149	7.0	0	1.95×10^{1}	2.82×10^{-1}	0	0	5.23×10^{2}	2.49×10^{1}	1.64	9.03×10^{-3}
0.210	9.6	4.5×10^{-4}	2.51×10^{1}	5.19×10^{-1}	1.89×10^{-5}	0	1.10×10^{3}	4.53×10^{1}	3.90	7.05×10^{-2}
0.300	15.2	5.7×10^{-3}	3.0×10^{1}	7.50×10^{-1}	9.95×10^{-4}	0	2.36×10^{3}	8.91×10^{1}	9.56	3.74×10^{-1}
0.420	21.0	1.49×10^{-2}	1.95×10^{1}	9.47×10^{-1}	5.30×10^{-3}	0	2.59×10^{3}	1.50×10^{2}	1.58×10^{1}	1.23
0.590	17.8	1.38×10^{-2}	7.50	7.91×10^{-1}	1.06×10^{-2}	0	1.28×10^{3}	1.47×10^{2}	1.47×10^{1}	1.73
0.840	13.0	6.45×10^{-3}	3.22	9.24×10^{-1}	2.49×10^{-2}	0	4.52×10^{2}	1.27×10^{2}	1.13×10^{1}	1.61
1.190	9.0	2.25×10^{-3}	1.86	1.13	6.69×10^{-2}	2.31×10^{-4}	1.57×10^{2}	9.03×10^{1}	9.1	1.40
		4.35×10^{-2}	1.11×10^{2}	5.37	1.10×10^{-1}	2.31×10^{-4}	8.06×10^{3}	6.78×10^{2}	6.61×10^{1}	6.42

† Zero denotes all rates less than 10^{-6} $kg \cdot sec^{-1} \cdot m^{-1}$.

Table 3.—*Summary of sediment characteristics of transported load*

	$U^{*\prime} = 3.14$ (cm·sec^{-1}) d_{65} (mm)	$U^{*\prime} = 20.1$ (cm·sec^{-1})		$U^{*\prime} = 63.8$ (cm·sec^{-1}) d_{65} (mm)			
	0.55	0.55	2.0	0.55	2.0	5.0	10.0
Transported $\bar{V}_m/\bar{V}_q$	8.63/6.36 = 1.36	3.8/2.94 = 1.29	7.30/6.72 = 1.05	4.70/4.0 = 1.18	5.1/6.7 = 0.76	6.2/6.86 = 0.90	10.0/9.1 = 1.10
Population % magnetite	0.4	3.8	0.4	7.0	3.5	1.4	0.1

Original population = 5.94/6.46 = 0.92

friction velocity important for grain transport:

$$U_*' + U_*'' = \sqrt{S_e R g}, \qquad (4)$$

where S_e is the energy slope (slope of the energy grade line), R is the hydraulic radius, and g is the gravitational acceleration. At this $U^{*\prime}$, very low bedload transport rates occur (on the order of 10^{-5} kg·sec^{-1}·meter^{-1}) and no magnetite moves in suspension (Table 2). The ratios of transport rates of quartz to magnetite range from 61 for the 0.3 mm size, to 927 for the 0.59 mm size (Table 2). The 0.21 mm size has a transport ratio approaching infinity as the magnetite transport rate approaches 0. Neither magnetite nor quartz is transported in sizes less than 0.149 mm. Thus, through time the lag deposit becomes enriched in magnetite in the sizes between 0.149 mm and 0.59 mm.

The size distribution of the transported material was calculated by plotting the unit transport rates for each fraction as cumulative weight percents of the total transport rate. The graphic mean transport size was then defined as $(\phi_{16} + \phi_{50} + \phi_{84})/3$. The graphic means are 0.31 mm for magnetite and 0.416 mm for quartz giving a constant terminal settling velocity ratio of magnetite to quartz of 1.36 (Table 3; Fig. 5). That is, the magnetite and quartz are more nearly equal in size than in the parent distribution whose settling velocity ratio is 0.92. At the same time the lag population develops a settling velocity ratio of less than 1 as coarser quartz grains make up an increasing amount of the bed. The concentration of magnetite in the transported population is 0.4% whereas the concentration in the initial population is 10% (Table 3 and Fig. 4).

For this case of $U^{*\prime} = 3.14$ cm·sec^{-1}, and for roughnesses greater than 2 mm, the transport rates of both magnetite and quartz are less than 10^{-6} kg·sec^{-1}·meter^{-1}. These rates are considered too small to analyze further.

At an energy slope equal to 0.01, $U^{*\prime} = 20.1$ cm·sec^{-1} and all available sizes of both minerals move in both bedload and suspension (Table 2). At a roughness size of 0.55 mm, which is the d_{65} of the initial distribution, the quartz moves at 8.6 to 217 times the rate of the magnetite over a range of sizes from 0.105 to 0.59 mm. The mean constant terminal settling velocity of the transported magnetite is 3.8 $cm \cdot sec^{-1}$ and that of the quartz is 2.95 $cm \cdot sec^{-1}$. Again, compared to the initial distribution, the mean sizes of the magnetite and quartz are more nearly equal. The concentration of the magnetite in the transported material is 3.8% (Table 3), a greater percentage than at the lower $U^{*\prime}$, but still less than the 10% of the initial distribution.

With the roughness increased to 2 mm, the mean settling velocity of the transported magnetite increases to 7.3 $cm \cdot sec^{-1}$ and the quartz to 6.72 $cm \cdot sec^{-1}$. Thus, the sizes are becoming more nearly hydraulically equivalent and closer to the mean sizes in the original distribution. But the concentration of magnetite in the transported sediment drops to 0.4%.

At a roughness of 5 mm, no magnetite is transported; only quartz sizes larger than 0.149 mm move. The lag deposit experiences the maximum possible enrichment seen thus far, and the magnetite mean size becomes larger than the quartz mean size.

At a roughness of 10 mm, only quartz sizes larger than 0.84 mm are transported. The amount of enrichment in the lag decreases and the mean settling velocities move towards the values of 4.76 $cm \cdot sec^{-1}$ for quartz and 5.67 $cm \cdot sec^{-1}$ for magnetite (assuming all sizes larger than 0.84 mm are removed).

At an energy slope of 0.1, corresponding to $U^{*\prime} = 63.8$ $cm \cdot sec^{-1}$, and with a roughness of 0.55 mm, the transport ratios are less than at lower $U^{*\prime}$, ranging from 3.7 to 298 over the sizes 0.105 to 0.59 mm (Table 3). This is not surprising because grains are no longer hidden, and the flow is less selective. The ratio of settling velocities of magnetite to quartz is 4.70 to 4.0 $cm \cdot sec^{-1}$ or 1.18, and the concentration of magnetite in the transported material is 7.0%. The mean sizes of magnetite and quartz traveling in the flow have become finer compared to the initial distributions but both sizes are coarser than the mean sizes traveling at a $U^{*\prime}$ of 20.1 $cm \cdot sec^{-1}$. Brady and Jobson (1973) and Minter and Toens (1970) have described qualitatively similar results in flume experiments.

With increasing roughness, the mean settling velocities increase for both the transported magnetite and quartz, and the ratios of settling velocities increase from 0.76 for $d_{65} = 2$ mm to 1.10 for $d_{65} = 10$ mm. At the same time the concentrations of magnetite in the transported material decrease from 3.5% for $d_{65} = 2$ mm to 0.1% for $d_{65} = 10$ mm. Qualitatively the lag deposit shows the reverse trends.

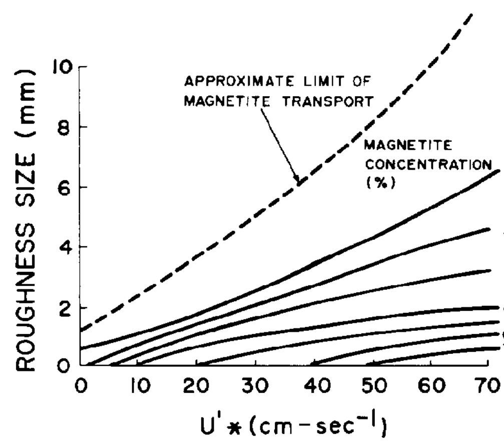

FIG. 4.—Predicted magnetite concentrations in the transported sediment for the case study described in the text. Concentrations increase with increasing $U^{*\prime}$ and decrease with increasing roughness.

To summarize, for any one $U^{*\prime}$ and for both densities, the transport rates for all sizes in the initial distribution decrease with increasing roughness. This is due to the factors Y, predicting the lift on a particle, and ξ, accounting for the natural interference among bed particles. The decrease is not uniform however, being greater on the finer tail of the distribution. This corroborates the analysis of critical entrainment summarized earlier (Slingerland 1977), in which finer and denser particles were argued to be more difficult to entrain because they have larger reactive angles through which they must be rolled, and also because they project lower into the velocity profile than the surrounding roughness elements.

The concentration of magnetite transported in the flow increases with increasing $U^{*\prime}$ for each roughness (Fig. 4) and decreases with increasing roughness for each $U^{*\prime}$.

At any one $U^{*\prime}$ and for both quartz and magnetite, increasing roughness shifts the highest transport rates toward the larger sizes. Thus the mean size of the lag deposit (excluding roughness elements) decreases with

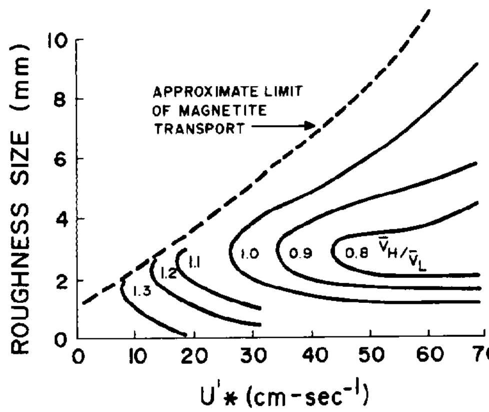

FIG. 5.—Predicted mean settling velocity ratios of magnetite to quartz in the transported sediment for the case study described in the text. Ratios decrease with increasing U*' and decrease and then increase with increasing roughness.

increasing roughness size. The opposite occurs with increasing U*' at any one roughness.

The settling velocity ratios of magnetite to quartz in transport decrease with increasing U*' for any one roughness (Fig. 5). For any one U*', the ratios first decrease then increase with increasing roughness. The ratios range from 1.36 at low U*' and roughnesses to 0.76 at high U*' and intermediate roughnesses.

The settling velocity ratios in the lag deposit are more difficult to predict. If no material is added to the site from upstream during any one entrainment event the size distributions available and the roughness will continually evolve. For example, to maintain the original sizes and hence settling relationships the transport ratios would have to bear the same ratios as the ratios of percentages of magnetite to quartz in the size classes of the initial distribution. That is, the transport ratios would have to be from 3.6 for the 0.105 mm class rising to 89 for the 0.59 mm class. But in general, at the lowest roughnesses, the transport ratios of quartz to magnetite are much higher, especially in the coarser sizes. Therefore the mean sizes of the magnetite and quartz will become more nearly equal and the settling velocity ratios will rise to become greater than 1.

This is consistent with solutions to a critical entrainment equation (Slingerland 1977) which showed that the highest settling velocity ratios in a deposit would occur when roughness sizes were on the same order as the heavy mineral sizes and the Boundary Reynolds Number was in the smooth to transitional regime. (The Boundary Reynolds Number for a U*' of 3.14, k_s = 0.55 mm, and υ = 0.01 is 17.3.)

EXPLANATION OF OBSERVED PLACER OCCURRENCES AND CHARACTERISTICS

The origin of the various placers listed in Table 1 can now be better understood in light of the four sorting processes. Three examples, one from each scale, will be discussed.

Bed Scale.—Heavy mineral concentrations commonly are observed on megaripple crests (Fig. 6), avalanche faces, and at the toes of tangential cross-strata formed from megaripple migration (Minter 1978; Smith and Minter 1980; Brady and Jobson 1973; Toh 1978; and McGowen and Groat, 1971). These concentrations are explained by assuming that the heavy mineral mean size is smaller than the light mineral mean size. On ripple crests (a of Fig. 6), the intensity of turbulence is at a minimum as a grain moves streamwise, whereas the local mean bed shear stress (as measured by U*) is at a maximum (Raudkivi 1967). Hence the horizontal drag force vector is larger than the vertical lift vector. Thus, larger and typically lighter grains are more easily rolled away than smaller, denser grains because the larger grains stick up higher into the velocity profile and their reactive angle is smaller. This leaves heavy and light grains of more nearly equal size. Any periodic turbulence is more likely to lift out the lighter grains and enrich the deposit further. This process is self enhancing because the roughness size moves towards the mean heavy size. And, as discussed in the entrainment and transport sections, $U*_c$ increases, independent of density, for grains near the roughness size. Thus more heavy and light minerals of the roughness size are trapped, continuing the enrichment process. A testable consequence of this explanation is that the settling velocity ratios of the heavy to light minerals should be greater than one in these deposits.

Heavy mineral accumulations in foresets of cross-strata (b of Fig. 6) can be explained in two ways. They can be the bases of shear-

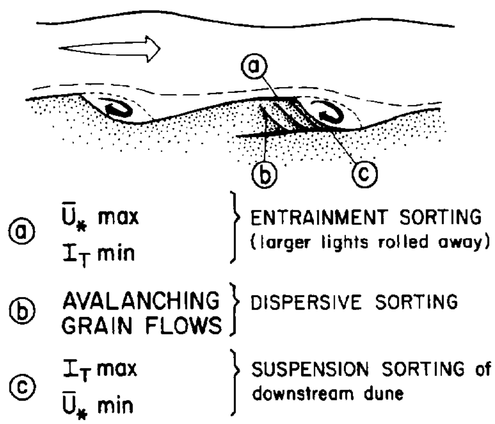

FIG. 6.—Heavy mineral enrichment at the bed scale: (a) crest of a dune, (b) foreset strata, (c) scoured bottom of trough. At (a) the time-averaged friction velocity is at a maximum compared to sites up- or down-stream, and the intensity of turbulence (I_T) is at a minimum. The reverse is true at (c).

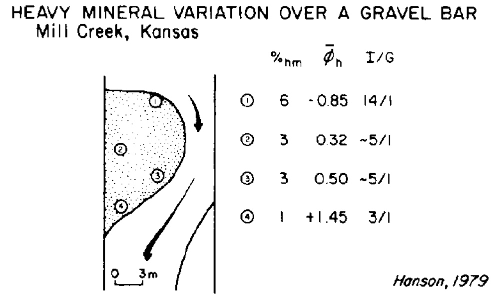

FIG. 7.—Heavy mineral enrichment at the bar scale, Mill Creek, Kansas. $\%_{hm}$ = percent total heavy minerals, $\bar{\phi}_h$ = mean size of heavy mineral fraction, I/G = ilmenite to garnet concentration ratio. The I/G ratio in the kimberlite source is 1/1.

sorted grain flows or they can reflect periods of erosion of the upstream, enriched dune crest as Smith and Minter (1980) proposed.

Accumulations at the toes of foresets and in troughs (c of Fig. 6) also have two possible explanations. First, at the point of reattachment of the flow, shear stress is at a minimum and turbulence is at a maximum. With dune migration, already heavily enriched deposits on the back of the adjacent downstream dune are reworked. As explained above, because the heavy and light grains there are more nearly equal in size, suspension sorting would leave a heavily enriched deposit of suspension-equivalent sizes. Second, suspension sorting, as explained above would lead to enriched size classes in the suspended load leaving the dune crest. The various classes would then settle to different points in front of the dune, with the finer and heavily enriched classes settling just upstream of the reattachment point.

These explanations amplify those of Brady and Jobson (1973) and McQuivey and Keefer (1969). Brady and Jobson explain crestal occurrences by arguing that heavy minerals "are selectively deposited from the moving bedload" (p. K-28) in the zone of deposition just downstream from the crest line. They also stated that foreset laminae are the result of segregations of heavy grains from the topset bed sliding down the lee slope. McQuivey and Keefer argued from flume experiments that both the relative intensity of turbulence and shear stress are low over a ripple crest and high in the trough. Using a Shield's type of entrainment function, they stated that both magnetite and quartz could be entrained in the trough because of high shear stresses but only quartz could be entrained at the low shear stresses on the crest. But their analysis of the variation in intensity of turbulence and mean bed shear stress appears to be wrong (Allen 1982). Thus, their explanation is seriously weakened.

Bar Scale.—Hanson (1979) studied the heavy mineral dispersion train in Mill Creek, a stream cutting across the Stockdale Kimberlite, Riley County, Kansas. The body in outcrop is 30 × 60 meters and occurs in flat-lying Permian shale, limestone, and dolostone containing a mature heavy mineral suite. Four 12-kg channel samples were collected on a small gravel bar (Fig. 7), one kilometer downstream from the kimberlite body. The samples were visually estimated by Hanson to be sandy gravel that decrease slightly in mean size from bar head to tail. A distinct decrease in heavy mineral concentration, size, and ilmenite to garnet ratio occurs downstream over the bar (Fig. 7). Thus this demonstrates well the miners' rule-of-thumb that bar heads are commonly enriched in heavy minerals.

The explanations for the concentration variation are, first, that heavy and light min-

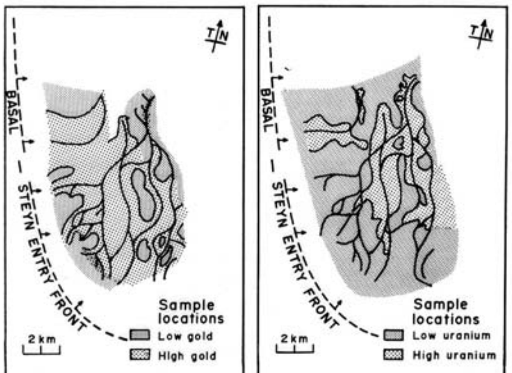

FIG. 8.—Heavy mineral enrichment at the system scale. The Steyn and Basal placers developed on humid alluvial fans in the Witwatersrand with high gold concentrations occurring sourceward of high uranium concentrations. This is a function of the sizes of light and heavy minerals available, and the average roughness sizes and friction velocities down the fans (modified from Minter 1978).

erals transported at high stage initially test the upstream roughness elements of a bar for stable positions. At constant $U^{*\prime}$, a higher percentage of heavy minerals would be trapped in the coarser bar top than in the sand bed of the channel (Fig. 4). Second, the head of the bar is continually eroded and transported to the tail. Because of the differential transport rates this provides a progressive enrichment. Third, the higher $U^{*\prime}$, due to flow convergence at the head, plots closer on the curve of concentration versus $U^{*\prime}$ and roughness (Fig. 4) to the optimum for lag enrichment. The coarser heavy sizes of the head are predicted by the transport model. As discussed previously, the mean size of a lag deposit increases with increasing $U^{*\prime}$, for a constant roughness size. The enrichment in ilmenite, a consequence of its greater density, occurs for similar reasons.

System Scale.—Minter (1978) described the paleoplacers in the Precambrian Upper Witwatersrand Group of South Africa. The Basal and Steyn placers of gold and uranium formed in braided river environments on wet fan deltas. Proximal pebble conglomerate comprising channel fills and longitudinal bars grades 20 km downslope into less deeply channeled quartz arenite built by shallow-water sand bars. Gold concentrations are highest in a strike-parallel band across the fan deltas between 4 and 10 km from the entry front (Fig. 8). The gold is most concentrated in pebble-supported conglomerate whose maximum clast size lies between 2 and 4 cm. Although the Basal and Steyn gold grain sizes are unknown, the gold sizes in the overlying B placer range from 0.5 to 0.005 mm, that is from medium sand to very fine silt (Minter 1978). The highest uranium values are displaced 2 km down paleoslope (Fig. 8). No uraninite grain size data are available.

As shown above, this spatial distribution of gold and uranium, at least in part, is controlled by the local sizes of heavy and light minerals available, the average roughness sizes at a site, and the long-term average U^* at a site. Generally, the clast size of humid alluvial fans and therefore the roughness decreases exponentially down slope (Rust 1979), as does $U^{*\prime}$, defined by $\sqrt{gRS_e}$. Then it is reasonable that given the source size distributions and the differential rates of heavy mineral comminution, a strike-parallel band would be formed where the ratio of local mean heavy size to local mean light size was appropriate for the local $U^{*\prime}$ and roughness. This point would be shifted downslope for uraninite because its lesser density would demand a smaller local mean $U^{*\prime}$. The location of the band within the paleodepositional system could be predicted if the initial settling velocity distributions of the gold were known, say from studies of the source veins. This could be coupled with sedimentological data on clast and slope decrease with distance from the source to provide inputs into the Einstein bedload function. This is in addition to other causes of concentration at a site such as a higher number of cut and fill cycles or erosion into a previously enriched local source.

CONCLUSIONS

A necessary condition for heavy mineral enrichment at any scale in streams is selective sorting by size and density due to differential entrainment, differential suspension, differential bedload transport, and shear sorting or kinetic sieving. The effects of differential entrainment, suspension, and transport on an initial distribution of medium-size quartz and 10% fine-size magnetite were modeled by solving the Einstein bedload function for expected ranges of grain friction velocities ($U^{*\prime}$) and bed roughnesses. For any one $U^{*\prime}$ (in the range approximately 3 to 63 $cm \cdot sec^{-1}$) and for both densities, the transport rates for all sizes in the initial distribution decrease with increasing roughness, the decrease being greatest on the fine tail of the distribution. The concentration of magnetite transported in the flow increases with increasing $U^{*\prime}$ for each roughness and decreases with increasing roughness for each $U^{*\prime}$. The settling velocity ratios of magnetite to quartz in transport decrease with increasing $U^{*\prime}$ for any one roughness. For any one $U^{*\prime}$, the ratios first decrease then increase with increasing roughness, ranging from 1.36 at low $U^{*\prime}$ and roughness to 0.76 at high $U^{*\prime}$ at intermediate roughness. Generally, these results are due to variations in the reactive angle of grains and the extent to which grains hide in the lower inner zone of the boundary layer.

Concentrations of heavy minerals on the backs of megaripples are due to larger and typically lighter grains being entrained and transported at a higher rate than smaller denser grains. This in turn is due to the locally higher mean friction velocity at the site. Heavy minerals are concentrated at the heads of bars because the bedload there first tests the roughness elements for stable positions, because of progressive enrichment as the bar slowly migrates downstream, and because flow convergence provides more optimal $U^{*\prime}$ on the curve of concentration versus $U^{*\prime}$ and roughness. Concentrations in bands parallel to depositional strike are a complex function of initial size distributions and the downstream variations in friction velocity and mean bed roughness.

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