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Stability analysis of a rejuvenated fluvial system

by

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with 5 figures and 1 table

Summary. Physical experiments by others have shown that a rejuvenated drainage basin does not simply respond by stream incision but hunts for a new equilibrium by incision, aggradation, and renewed incision. Overbank flooding and temporally fluctuating sediment yields are possible manifestations of this complex response. To better understand the feedback loops and magnitudes of variables necessary for this instability, we have conducted numerical experiments simulating fluvial response to base level lowering. The model consists of the partial differential equations describing unsteady, nonuniform flow and sediment transport over a mobile, granule-sized bed written for a trunk stream 200 km long with outflow water volume of 10 m^3/sec that experiences a 10 m base level drop at time 0. Tributaries enter every 20 km alongstream and provide sediment at rates proportional to a complex function of the cumulative changes in bed elevation at their junctions. The magnitude of sediment outflow for a given time history of changes in bed elevation (Q_{sie}) and the lag in response time of the tributary (w) are both made variable. Results show that a minimum Q_{sie}, equal to three times the sediment yield that would arise from simple channel entrenching alone, is necessary for a complex response. In addition, w must be greater than 200 years. Under these conditions oscillations in bed elevation occur along the entire trunk stream with upstream tributary junctions experiencing the most distinct episodes of cutting and filling. The interactions causing the oscillations appear to be between the sediment production of tributaries and the local sediment transport rates of the trunk stream as well as interactions among sediment transport rates at different along stream sites. This behavior should produce a set of several continuous terraces converging in the downstream direction.

Introduction

The occurrence of long-term river channel aggradation and degradation is a subtle but pernicious geomorphic hazard, leading as it often does, to overbank flooding, bank instabilities, arroyo development, and increased rates of sediment production. In the semiarid American west, several causal mechanisms have been postulated, including extrinsic factors such as climatic changes (EULER et al. 1979), land-use changes (HASTINGS 1959,

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PATTON & BOISON 1986), or tectonics (VOLKOV et al. 1967, BURNETT & SCHUMM 1983), and intrinsic factors such as complex responses (SCHUMM 1973, SCHUMM & PARKER 1973) or fluvial thresholds (BULL 1979, PATTON & SCHUMM 1975, 1981). Of these, perhaps the intrinsic factors, and complex responses in particular, deserve special attention, because until the full range of their effects is known, it will be difficult to ascribe a particular fluvial behavior to a particular extrinsic cause.

Complex response refers to the damped oscillation of variables in the fluvial system after an initial perturbation from equilibrium. The example most often given (SCHUMM 1973, SCHUMM et al. 1987) is the response of an experimental drainage basin when rejuvenated. Contrary to intuition, a new grade is reached only after many cycles of aggradation and degradation in the trunk stream. The trunk stream first responds to a drop in base level by incising its channel. As the wave of erosion migrates upstream, tributaries experience local base level drops of varying magnitudes and phase lags. The trunk stream, which initially was adjusting its slope, cross-sectional area, and so on, to transport a given sediment yield from its tributaries, now must transport more. Consequently, it aggrades and widens. This either slows or reverses the rate of base level drop experienced by the tributaries, their sediment production slows, and the trunk channel narrows and incises into the previously deposited alluvial fill. The cycle begins anew.

This behavior is most easily seen as fluctuations in sediment yield from the trunk stream (Fig. 1). The result of an initial base level drop is an initial rapid rise and exponential decline in sediment yield, followed by quasi-periodic rises and falls. From a systems point of view, the behavior is a stable one, but instead of returning to equilibrium

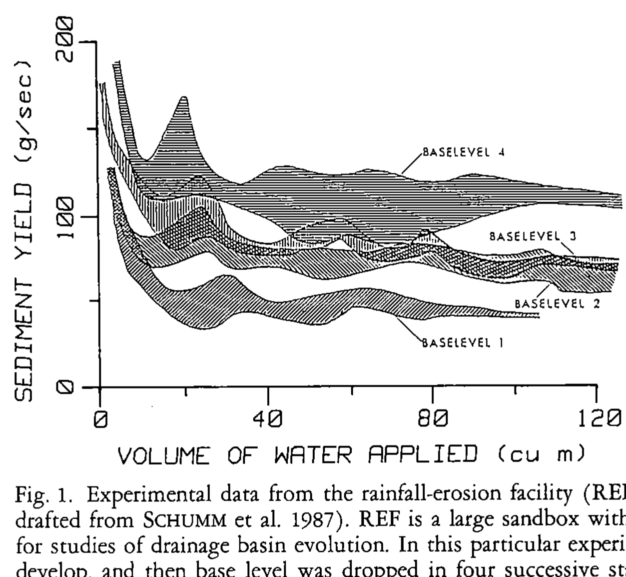

Fig. 1. Experimental data from the rainfall-erosion facility (REF) at Colorado State University (redrafted from SCHUMM et al. 1987). REF is a large sandbox with an overhead sprinkler system used for studies of drainage basin evolution. In this particular experiment a drainage net was allowed to develop, and then base level was dropped in four successive stages. The resulting sediment yields from the trunk stream are plotted versus volume of water applied (a surrogate for time from each base level drop), showing initial high yields followed by a damped harmonic decay towards steady state values. This is one manifestation of the complex response of SCHUMM & PARKER (1973).

asymptotically, the fluvial system returns through damped oscillations. Systems theory (see, for example, LEVINS 1974, SLINGERLAND 1981) would suggest for this case, that at each level of feedback, negative feedback outweighs positive, but negative feedback with long time lag is of appropriate magnitude compared to the shorter loop negative feedback to cause the oscillations (LEVINS 1974).

While the above description and systems explanation seem intuitively correct and applicable to natural systems, it would be reassuring to discover that the physics of the system, as captured in a mathematical model, produces the same results. Possibly the variations seen in the flume are an artifact of the distorted scaling or the peculiarities of sediment transport in very shallow flows. Also, it would be useful to define the important variables, the feedback loops, and the initial and boundary conditions that produce the ratios of negative feedback conducive to complex response.

Here we present a mathematical model of a fluvial system and use it to conduct experiments on complex response. The results of 9 simulations of system response to a simple base level drop at time zero, indicate that a time lag of greater than 200 years and significantly enhanced sediment outflows from tributaries are necessary and sufficient conditions for a complex response. Under these conditions, the trunk stream experiences slightly diachronous cutting-and-filling and oscillations of sediment yield that decay with time.

The model

The mathematical model describes quasi-two dimensional, unsteady, nonuniform flow through an erodible trunk stream with tributaries. Boundary conditions are steady-state sediment and water discharges from tributaries entering the trunk stream, water surface elevation at the stream mouth, and rates of grain size decrease and width increase downstream. The model is written in terms of three dependent variables, the cross-sectionally averaged flow velocity, u; water surface elevation, h; and mean bed elevation, b. The two independent variables are downstream distance, x; and time, t. The governing equations are:

— conservation of momentum for clear water

$$(1) \qquad \frac{\partial Wu(h-b)}{\partial t} + \frac{\partial Wu^2(h-b)}{\partial x} + gW(h-b)\frac{\partial h}{\partial x} + \frac{gn^2Wu|u|}{(h-b)^{1/3}} - \beta Q_i u = 0$$

— continuity of water

$$(2) \qquad \frac{\partial W(h-b)}{\partial t} + \frac{\partial Wu(h-b)}{\partial x} - Q_i = 0$$

— continuity of sediment

$$(3) \qquad \alpha\frac{\partial W(h_c-b)}{\partial t} - \frac{\partial WQ_s}{\partial x} + Q_{sic} + Q_{sie} = 0$$

— bedload transport equation

(4a) $$Q_s = 5\, d\, [(s-1)\, g\, d]^{1/2} P (\Theta^{1/2} - 0.7\, \Theta_c^{1/2})$$

(4b) $$P = \left[1 + \left(\frac{0.2668}{\Theta - \Theta_c} \right)^4 \right]^{-1/4}$$

— width equation

(5) $$W / W_{max} = 8 * SQRT (Q / Q_{max})$$

— tributary response equation

(6) $$Q_{sie} = N * \sum_{k=1}^{\infty} [B(k) * Q_{se}(k)]$$

where g is the acceleration of gravity, n is Manning's roughness coefficient (0.03 $ft^{1/6}$). Q_i is the lateral inflow of water (discharge per tributary), beta is a tributary momentum factor (here equal to 1.0), Q_{sic} is the lateral inflow of sediment supplied constantly by the drainage basin (sediment discharge per unit length), Q_{sie} is the extra sediment inflow produced as a response to the history of water surface lowering at a tributary junction (sediment discharge per tributary), alpha is one minus the porosity of channel sediment (here equal to 0.7), Q_s is the bed material transport in volume per unit time, B(k) is the amount of base level change that occurs in the kth time step after an initial base level drop, N is a sediment production coefficient, and Q_{se} (k) is the sediment discharge per meter of base level change per tributary.

The conservation equations (1–3) are the standard vertically integrated partial differential equations of mobile-bed, open channel flow. They are employed with the assumptions that suspended sediment loads will not alter the effective fluid density, the fluid pressure is hydrostatic, and the channel width-depth ratio is greater than 10 (that is, the channel is hydraulically wide).

The bedload transport equation (4) is due to ENGELUND & FREDSØE (1976). Critical Shields parameters are estimated from the Shields curve assuming quartz density particles.

The width equation (5) is derived from the empirical laws of hydraulic geometry (LEOPOLD & MADDOCK 1953).

The tributary response equation arises from a preliminary study of the sediment yield from a single channel with one-tenth the outflow discharge of the present trunk stream, one-fourth the length, and similar sediment size. Equations 1–5 were solved, subject to the boundary condition of different small base level drops, and the resulting sediment discharge recorded. This is the sediment discharge from simple channel entrenchment with no contributions from second order tributaries or side walls. The results (Fig. 2) suggest that the amount of added sediment outflow at any particular time is close to a simple, direct proportion of the amount of base level drop, and is a super-exponential function decreasing with elapsed time since the drop. Experimental confirmation can be found in BEGIN et al. (1981, runs 11 and 12) and GARDNER (1983) who studied knickpoint retreat in simple channels in flumes. They present longitudinal profiles through time after base level drop. If we assume sediment yield is directly proportional to the area difference be-

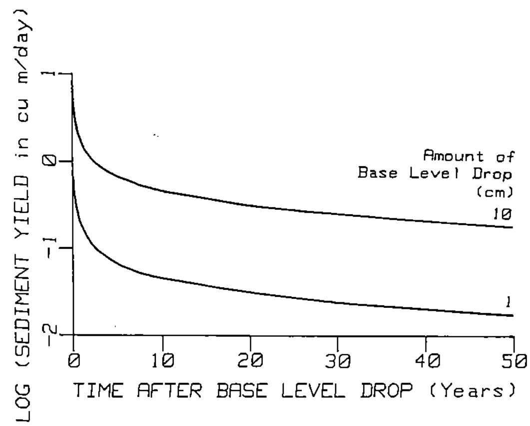

Fig. 2. Model results from an experiment simulating channel excavation of a graded stream with no tributaries. After a base level drop, sediment yield decays super-exponentially (note the logarithmic scale). Different magnitudes of drop shift the curves vertically but do not alter their form, suggesting magnitude of sediment outflow at any time is linearly proportional to the amount of base level drop.

tween successive profiles (i.e., no sediment storage), then sediment yield is linearly proportional to the amount of base level drop and a complex function of time ($t^{-3}\exp(-1/t)$) (BEGIN et al. 1981, equation 24). In our study, the results of the preliminary run have been normalized to values of sediment discharge per meter of drop, and coded into a vector of outflow values, $Q_{se}(k)$, where k equals the number of timesteps after the base level drop. The tributary sediment outflow at any time step k then is the sum of sediment outflows due to all past base level drops or rises. Base level rises are incorporated by allowing B(k) to take on negative values. However, tributary sediment inflow never ceases completely in these simulations. To increase the amount of sediment outflow a tributary produces above simple channel entrenchment, we multiply all B values by the sediment production coefficient N, greater than one. To simulate a lag time in tributary response, the time step counter of variable B is adjusted accordingly.

In collecting a set of controlling variable values, we have used general characteristics of natural streams in the midwestern United States (SNOW & SLINGERLAND 1987), modified for greater simplicity and to favor semi-arid conditions. The trunk stream is arbitrarily taken to be 200 km long, with tributaries entering every 20 km alongstream. The tributaries are identical, each with 1 m^3-sec^{-1} constant water outflow, giving the trunk stream an outflow volume of 10 m^3-sec^{-1}. Sediment outflow is also entirely supplied by the tributaries, and the constant component of sediment outflow for each is 0.0006 m^3-sec^{-1}. Trunk stream gradients range from 0.004 upstream with a bed sediment mean size of 6 mm to 0.001 downstream and bed sediment size of 2 mm. It should be noted that the model does not compute this downstream variation in size, it responds to it as a boundary condition to produce the graded slopes. Tributary gradients range from 0.005 upstream to 0.0025 downstream. Trunk stream water velocities are about 0.7 m-sec^{-1}, and depths increase downstream from 0.17 m to about 0.54 m. Equations 1–5 and the above boundary and initial conditions constitute a closed set of equations. These are solved in the FORTRAN program COMPLEX using a finite difference scheme due to FREAD (1978).

Nine combinations of input variables have been chosen to explore the system's behavior (Table 1). Run A1 is designed to represent the system response to a small base level drop. Tributaries increase their sediment outflow as soon as their local base level is

Table 1 Summary of conditions for the numerical experiments.

Experiment	Amount of Base level Drop (m)	N of Equation (6)	Lag Time (yrs)
A 1	2	1	0
B 1	10	1	0
B 2	10	1	20
C 1	10	1	200
C 2	10	10	200
D 1	10	3	0
D 2	10	3	120
D 3	10	3	200
D 4	10	3	300

lowered and contribute only the sediment eroded out of their channels. Run B 1 is similar to A 1 but with a 10 m base level drop on the trunk stream. Run B 2 introduces a 20 year time lag in tributary response. Run C 1 is the same as the B series but introduces a 200 year time lag. C 2 is similar to C 1 but multiplies the tributary sediment outflow by ten, thus crudely simulating gulley development and channel slumping in the tributary subbasin. The D series maintains the 10 m base level drop, fixes the sediment outflow multiplier at three, and varies the time lag from 0 to 300 years.

Experimental results

Experiment A 1 produces a smooth, seemingly exponential decrease of sediment yield with time from the trunk stream (Fig. 3). The B series produces similar results, with no significant difference between B 1 (no time lag) and B 2 (20 years lag). The C series begins to exhibit oscillatory variation in sediment discharge with time. C 1 shows a slight rise at about 400 years after base level drop and C 2, in which tributaries produce 10 times the sediment outflow as in run C 1, exhibits wild oscillations that terminate the computation.

Based on these preliminary runs, the D series was created using a sediment production coefficient N, of 3. The results of these runs (Fig. 4) demonstrate that with increasing time lag, the perturbations in sediment yield grow in amplitude and period. A time lag of 120 years (D 2) produces a single oscillation whereas a time lag of 200 years (D 3) produces an oscillation at 250 years, a relatively smooth decline to about 2000 years, and then a set of oscillations which period is about 500 years. A time lag of 300 (D 4) years produces a sediment outflow graph with broad, somewhat irregular oscillations which period is about 800 years.

A plot of bed erosion at tributary junctions over the duration of experiment D 4 (Fig. 5) reveals cut-and-fill cycles on the order of decimeters to about 7 meters occurring with a period of about 800 to 825 years. The first cycle of cut-and-fill moves upstream as a wave but all subsequent cut-and-fill cycles start upstream and move downstream. Surprisingly, upstream sites experience the greatest amplitudes; these decay with time and distance downstream.

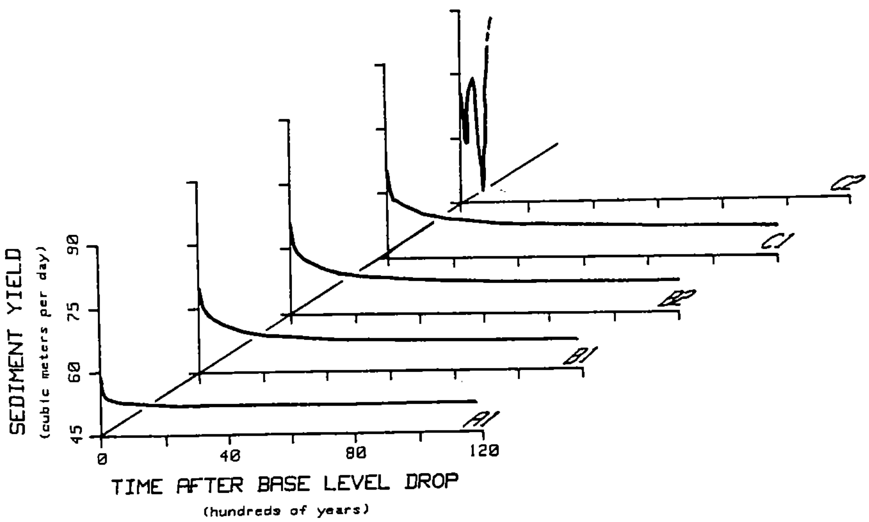

Fig. 3. Computed sediment yields from a trunk stream that experiences a base level drop at time 0. Experimental conditions are given in Table 1; see text for interpretation.

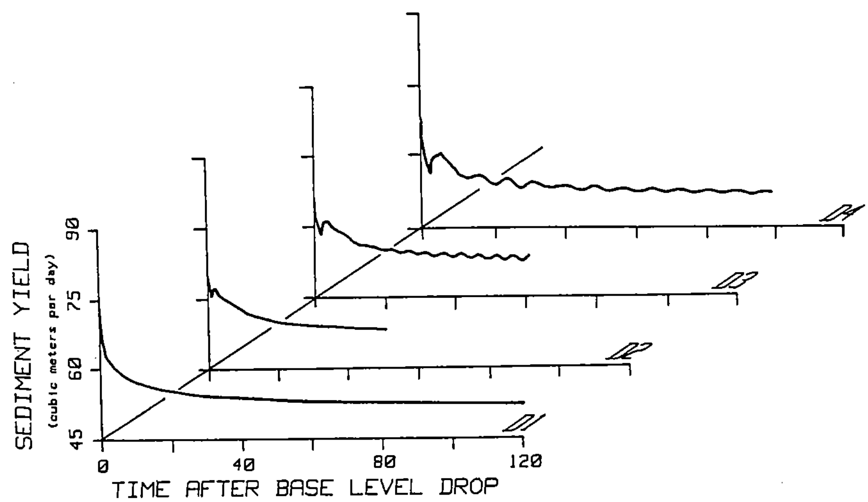

Fig. 4. Computed sediment yields from a trunk stream that experiences a base level drop at time 0. Experimental conditions are given in Table 1. Note that experiments D3 and D4 in which the lag times for sediment production from the tributaries are 200 and 300 years, respectively, produced oscillations in sediment yields that decay with time similar to the physical experiments of Figure 1.

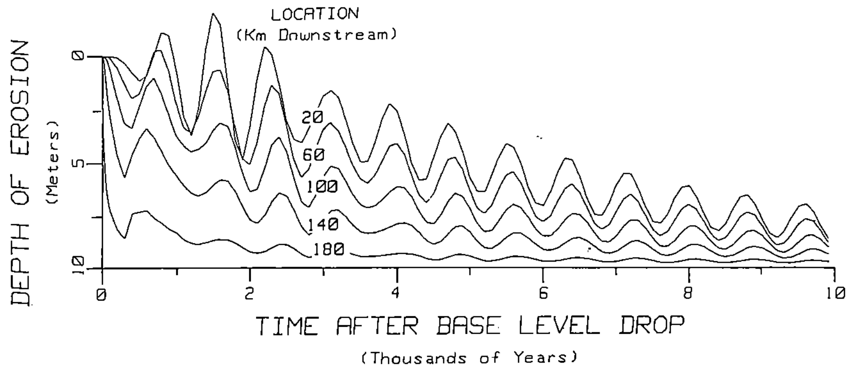

Fig. 5. Depth of erosion in the trunk stream at five tributary junctions following a base level drop (Experiment D4). A wave of cut-and-fill migrates upstream over the first 1000 years, after which waves migrate downstream over the next 9000 years.

Discussion

The results of experiment D4 (Fig. 4) are remarkably similar to those of SCHUMM & PARKER presented in Fig. 1. An initial sharp decline in sediment production is followed by a spike and then decaying, irregular oscillations. It appears that threshold phenomena such as bank slumping are not necessary conditions for this behavior because these are not incorporated in our mathematical model. Most likely, the cause lies in feedback loops between tributary sediment production and local sediment transport in the trunk stream near each individual tributary mouth, and among local sediment transport rates along the trunk stream, because both an imposed time lag and extra sediment outflow from the tributaries are needed to cause the complex response.

Consideration of Fig. 5 suggests the following explanation. During the first 300 years before any tributaries produce excess sediment, cutting occurs by simple knickpoint retreat. A wave of erosion migrates approximately 180 km upstream. At 300 years (the time lag in this series), the downstream tributaries begin to contribute sediment to the trunk stream and the trunk stream aggrades, more so upstream than downstream. The upstream reaches must aggrade more because they must outpace aggradation downstream to obtain an increased slope. The wave of aggradation sequentially turns off sediment production in the tributaries, thus initiating erosion again. This erosion starts first in the downstream portions of the trunk stream, but the upstream portions cut quicker and deeper. We feel this more extreme response upstream is because the larger sediment sizes and smaller magnitudes of water discharge and stream width require more extreme changes in slope to accommodate a given percentage increase in sediment load. As a result of this incision, upstream tributaries begin producing sediment, and a wave of alluviation sweeps downstream thus continuing the oscillations.

Conclusions

These results suggest that the phenomenon of complex response may well be an inherent property of a fluvial drainage system that arises when the magnitude and time lag of sediment input from tributaries are large with respect to the sediment transport capacity of the trunk stream. The exact feedback mechanisms are still not clear, embedded as they are within the complexities of the differential equations, but the two likely candidates are the feedback between sediment production of tributaries and the local base level in the trunk stream, and the sediment transport along the trunk stream as a response to the differing local base levels.

As others have stated, the bed elevation oscillations discussed above would produce a set of several continuous terraces along the main stream that would converge in the downstream direction. In addition, our results show that the age of any particular terrace surface would become younger downstream, although the total time range would be a fraction (in this series, about three-tenths) of the oscillation period.

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